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Applications of reliability theory in various contexts

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ABSTRACT

Reliability theory has grown in the last six decades into an independent discipline by drawing tools from several areas including mathematics, statistics probability theory, and actuarial science. Over the span of tackling its issues, reliability examination have contributed a few ideas and hypotheses to numerous different subjects. These themes have brought about opening up new zones of examination in a few fields of logical movement. This paper is dedicated to recognizing a portion of the uses of the thoughts, ideas and results in reliability theory to different parts of learning. Since the utilization of dependability ideas to various settings is too tremendous to even consider surveying, the creator has exhibited just some key controls where the collaboration has evoked generous intrigue.

1. Introduction

Reliability concepts have also found many useful applications in the construction of income distributions, analysis of human settlements, under-reported incomes, and bibliometry. An overview of some imperative applications is exhibited here. Accordingly, a talk of the essential ideas and philosophy in reliability theory accept significance in a plenty of territories of logical action [1].

In the greater part of the exchanges on reliability theory, the lifetime is treated as persistent. The demonstrating and examination perspectives use disseminations and procedures that regularly speak to a non-negative ceaseless arbitrary variable. Relatively [2], significantly less writing is accessible when the lifetime is discrete. Be that as it may, there are some convincing motivations to consider disappointment times as discrete irregular factors taking on non-negative whole number qualities. At the point when a bit of gear works in cycles and the perception is the quantity of cycles finished before disappointment, the lifetime is plainly discrete. So additionally is the situation when the gadget is checked just in finished units of time, similar to what number of disappointments have happened toward the consummation of 60 minutes, two hours, etc [3].

The absence of exactness of estimating gadgets may likewise produce discrete lives. There are events to incline toward checks over clock time notwithstanding when the last is accessible. In weapons reliability, the quantity of rounds shot could really compare to the age at disappointment. The equivalent is the situation with lifetimes of vehicle tires wherein the quantity of kilometers keep running before it winds up out of utilization is wanted to the quantity of days before disappointment. These kinds of issues give a solid driving force to examining dependability [4].

Therefore, the regions of study considered here are survival examination, sociologies, hazard theory, data theory, science and insights. Indeed, even in this constrained article here, the analysis is not exhaustive and is limited to a few illustrative cases [5].

2. Survival Analysis

Survival analysis describes a body of statistical procedures for analyzing data on a random variable X that represents the waiting time until the occurrence of an event of interest. In the terminology of survival analysis, the concerned event is referred to as death and the waiting time as survival time. The basic concepts used here also are the survival function S(x) and hazard rate function h(x), which bear the same relationships and properties as in reliability theory. Methods of analyzing discrete time survival data have been considered by many authors. They have pointed out the advantages of this approach as [6] suited to the analysis of longitudinal data, it can handle time-variant models and time-variant predictors, model violation can be easily checked and corrected, it is easier to handle censored observations.

It is appropriate when duration time is grouped or banded into discrete intervals, in which case the data can be truly discrete [7,8].

3. Additive Hazards Model

The PH models assume multiplicative relationship with the baseline hazard. There are situations in which an additive relationship is more meaningful in which case we have an additive hazards model. For example, in tumorogenic- ity experiments, when the dose effect on tumor risk is of interest, the excess risk becomes an important factor. Clinical trials that seek the effectiveness of treatments often experience lag times of treatment effectiveness after which treatment procedures will be in full effect. If X is the lifetime under study and Y is another random vari- able, both discrete, from which information on X can be gathered, the additive hazards model assumes a structural relationship of the form [9,10]

$$h(x|Y=y)=a(y)+h(x),\ x,y=0,1,2,\ldots.$$

4. Proportional Odds Model

The notion of proportional odds model was introduced and studied by Bennett (1983), Kirmani and Gupta (2001), and Rossini and Tsiatis (1996). They argued that such a model is motivated by the following facts [11,12]:

In the PHM, $h_i(x)/\ h_0(x)$ is constant in x, thus ruling out situations in

The assumption of constant hazard rates ratio becomes unreasonable when initial effects such as differences in the stage of disease or in treatment disappear with time;

Demonstration of the effectiveness of a cure on mortality of a disease group compared to that of a disease-free control group requires that the hazard ratio converge to unity [13].

5. Social Sciences

In this section, we deal with the impact of reliability concepts in various branch random variable under consideration in these areas, but non-negative random variables where the notions of reliability are logical and interpretable. Most of the applications in economics is in the context of income analysis. In the early literature on income distributions, several discrete distributions were proposed [14,15].

As special cases, we have the Waring and Yule distributions mentioned in Paper 3. Recently, modelling incomes is performed with the aid of contin- uous distributions like various Pareto forms, log-normal, beta, gamma, Weibull, Singh-Maddala, Dagum, Pareto-lognormal, etc. Quantile functions can also be employed to describe the patterns in income variation as is done in Tarsitano (2004) and Haritha et al. (2008) [16,17].

Zenga (2007) proposed a curve to assess inequalities in income as

$$A(x) \quad 1 \quad \frac{E(X|X \le x)}{E(X|X > x)}, \qquad (2)$$

and pointed out that A(x) satisfies several desirable properties for a discrete data set. The relationship A(x) has with various reliability functions can be easily written down. Nair et al. (2012a) studied some properties of the curve A(x) of which those related to reliability concepts are given below:

$$= A(x) \quad 1 \quad \frac{x + r(x)}{m(x) + x}$$

$$= \frac{m(x) + x - \mu}{(x)(m(x) + x)} F$$

$$= \frac{\mu - x + r(x)}{\mu - F(x)(x - r(x))}.$$

The three different curves mentioned above are interrelated and each can be used to compare income distributions vis-a-vis the extent of inequality among them. All those aspects are studied in Pundir et al. (2005) and Nair et al. (2012a) [18].

Another important class of inequality measures are called measures of poverty. These are based on a poverty line and concentrate on proportion of individuals below that line. Similarly, we can talk about affluence indices by considering the proportion of individuals in the population above an affluence line. An important problem in business activities is concerned with the pricing of a new durable product. Normally, the price as well as the sales of the product decline over time, irrespective of the discounts declared and the length of the finite planning horizon. Models are built up to relate the price and demand in which the hazard rate function plays a principal role. Bass (1987) proposed a model in which sales can increase to a peak and then decline. A model with a closed-form solution to this problem has been suggested by Sethi and Bass (2003). Suppose a customer buying the product at time x given that he has not bought it until now depends on the product price p(x), the demand rate D(p(x)) and F (x), the probability that a customer bought it by time x. Then the model prescribed is [19,20]

$$h(x) = D(p(x)), \qquad (3)$$

where $D(t) = a - bt$.

In political science, two important topics in which coherent structures find application are the famous *impossibility theorem* of Arrow (1951) and voting games. Of these, Arrow's theorem can be stated in verbal form in the following manner. When voters have three or more distinct options, no rank over voting system can convert the ranked preferences of individuals into a community wide ranking. For the validity of the theorem, the preference criteria of the voting shall be such that [21]

if every voter prefers alternative X over Y , then the group prefers X over Y;

if every voter preference between X and Y , is unaltered the group prefer- ence should also be unaltered;

no single voter is allowed the power to always determine group preferences, regardless of the other individual preferences.

Although the classical proof of Arrow's theorem is based on properties of com- plete pre-orders, Pechlivanides (1975) have given an alternative proof employ- ing coherent-structure arguments in reliability theory. An exhaustive discussion of Arrow's theorem and a mathematical proof in terms of coherent structures is given in Bhattacharjee (1988) [22].

6. Risk Analysis

Generally, risk refers to the underlying uncertainty of a given course of action, but is defined differently in different circumstances in which it is confronted. In finance, risk analysis refers to the uncertainty of forecasted cash flows, vari- ations of portfolios and stock returns. Several measures of risk have been pro- posed in literature that focus on extreme and high consequence events. Financial risk measures are often interpreted as the amount of money to be held in reserve for a risky investment. A commonly used risk measure is the *value at risk* de- fined as [23]

$$VaR_X(p) = F^{-1}(p), \qquad (4)$$

where $F^{-1}(p)$ inf x F (x) p , $0 < p < 1$, is the quantile function of the random variable X. In the discrete case, one takes $F^{-1}(p)$ as the smaller integer value for which $F(x) \ge p$. We interpret VaR_p as the maximum risk, for a given time horizon, experienced by 100 p% of cases. It is also the minimum amount of capital that would be added to a risky instant so that the probability of loss does not exceed p. Further refinements to

VaR gives certain other measures that have close proximity to reliability functions [24].

Singpurwalla (2006) has discussed a parallel connection between reliability con- cepts with present value of a risk-free bond and the interest rates. A risk-free coupon assures the holder $1 after time T from the time of purchase of the bond. If the bond is purchased at time t and it is held till time t + T , the pur- chase price P (t, T) is known as the present value of the bond at time t. We have P (t, 0) = 1 and P (t, T) decreases in T . The present value P (t, T) depends on the interest rate r(s) which changes with time. Then, Singpurwalla (2006) has argued that P (t, T) has the same behaviour as that of a survival function and that of r(s) as the case of a hazard rate. His arguments can be translated into discrete time as well. He has also developed the notions of IIR (increasing inter- est rate), IAIR (increasing average increasing rate), NWO (new worse than old) which are similar to the IHR, IHRA and NBU concepts in reliability [20].

We have provided only a few selected instances of the interface between reli- ability and risk. For a detailed study of these and other prospective connections between the two disciplines, we refer to Straub (1970), Denuit et al. (2005), and Zio (2007). Compound distributions and equilibrium compound distribu- tions have an important role in representing stop-loss premiums of interest in connection with insurance claim modelling; see Willmot et al. (2005).

7. Information Theory

In the conventional approach, the life distribution is characterized by reliability functions such as hazard rate, mean residual life, etc., and the ageing properties of a device are described in terms of the behaviour of these functions. Ebrahimi (1996) introduced an alternative methodology by characterizing the lifetime dis- tribution in terms of Shannon's measure of uncertainty and that classes of life distributions, which are different from these based on reliability functions, can be obtained in such an approach. This work has stimulated a lot of research during the past decades, wherein various measures of uncertainty based on the residual life distributions have been proposed. In this section, we review some of the important results that concern discrete models [22].

An extensive literature is available on different types of measures of uncertainty and their dynamic versions in the continuous case for which discrete analogues are yet to be found. The works of Kumar and Taneja (2011), Navarro et al. (2011), Abbasnejad (2011), Park et al. (2012), and Chemany and Bharatpour (2014) and their references give a good account of the recent developments on these topics.

8. Mathematics and Statistics

In this section, we highlight the impact of the ideas developed in reliability in some other disciplines. Mathematics and statistics richly benefited by various tools used in the course of solving problems in reliability analysis such as total positivity, Polya frequency functions, association of random variables, renewal theory, majorization, Schur functions, and so on. Boland and Proschan (1988) have discussed these aspects in detail. Generally, a function K(x, y) of two real variables, ranging over a linearly ordered sets X and Y , is totally positive of order r(T P_r) if

$$\begin{vmatrix} K(x_1, y_1) & K(x_1, y_2) & \cdots & K(x_1, y_m) \\ K(x_2, y_1) & K(x_2, y_2) & \cdots & K(x_2, y_m) \\ \vdots & \vdots & & \vdots \\ K(x_m, y_1) & K(x_m, y_2) & \cdots & K(x_m, y_m) \end{vmatrix} \geq 0 \qquad (5)$$

More applications in other areas like mathematical biology, approximation the- ory, and operator theory can be seen in Gasca and Micchelli (1996), Gori and Pitolli (2002) and Marshall et al. (2011) [23,24].

Some results on majorization, Schur-convexity and Schur-constancy were discussed in connection with the Bayesian concepts of ageing in Papers 7 and 8. Subsequently, their applications have spread over many areas in math- ematics like combinational analysis, geometry, matrix theory and numerical analysis; for details, see Marshall et al. (2011). Association of random variables has triggered many interesting contributions in statistical theory. In its strongest version, $\mathbf{X} = (X_1, \ldots, X_p)$ is said to be associated [25].

Thus, the result says that if f and g are increasing or both decreasing, then the association is positive, and they are negatively associated otherwise. This result is fundamental in statistical mechanics, probabilistic combinatorics and random graphs wherein many increasing events are interpreted to be positively correlated. Total positivity is the strongest in a large hierarchy of association concepts and therefore it serves as a sufficient condition to establish positive association in terms of other weaker concepts which are sometimes difficult to prove[26].

9. Conclusion

Renewal theory plays an important role in various aspects of reliability theory. In order that a device or system is able to perform its intended function without disruption due to failure, several types of maintenance strategies are spelt out in reliability engineering. Two such strategies are age replacement policy and block replacement policy.

Apart from reliability, equilibrium models have relevance in formulation of maintenance policies, income analysis, and insurance. Many of the findings mentioned above do not have their discrete analogues due to slow progress in paving theoretical foundations in discrete-time reliability modelling. Much work still needs to be done in this regard

References

1. Asha, G., 1995. Some Bivariate Lifetime Models in Discrete Time. Ph.D. thesis. Cochin University of Science and Technology, Cochin, India.
2. Bain, L.J., 1974. Analysis for the linear failure-rate life-testing distribution. Technometrics 16, 551–559.
3. Bain, L.J., 1978. Statistical Analysis of Reliability and Life-

Testing Models. Marcel Dekker, New York.
4. Belzunce, F., Candel, J., Ruiz, J.M., 1998. Ordering and asymptotic properties of residual income distributions. Sankhya¯, Series B 60, 331–348.
5. Bennett, S., 1983. Analysis of survival data by proportional odds model. Statistics in Medicine 2, 273–277.
6. Engineering and Informational Sciences 16, 67–84.
7. Cai, J., 2001. Discrete Time Risk Models Under Stochastic Forces of Interest. Ph.D. thesis. Univer- sity of Melbourne.
8. Clayton, D.G., 1978. A model for association in bivariate life tables and its application in epidemi- ological studies of familial tendency in chronic disease incidence. Biometrika 65, 141–151.
9. Conway, R.W., Maxwell, W.L., 1962. A queueing model with state dependent service rates. Journal of
10. Doray, L.G., Luong, A., 1995. Quadratic distance estimators of the zeta family. Insurance: Mathe- matics and Economics 16, 255–260.
11. Doray, L.G., Luong, A., 1997. Efficient estimators of the Good family. Communications in Statistics, Simulation and Computation 26, 1075–1088.
12. Ebrahimi, N., 1986. Classes of discrete decreasing and increasing mean residual life distributions.
13. Fox, W.R., Lasker, G.W., 1983. The distribution of surname frequencies. International Statistics Review 51, 81–87.
14. Gasca, M., Micchelli, C.A., 1996. Total Positivity and Its Applications. Kluwer Academic Publish- ers, Boston, MA, pp. 133–155.
15. Gori, L., Pitolli, F., 2002. On some applications of a class of totally positive bases. International Conference on Wavelet Analysis and Applications 25, 109–118.
16. Guess, F.M., Park, D.H., 1988. Modelling discrete bathtub and upside-down bathtub mean residual life functions. IEEE Transactions on Reliability 37, 545–549.
17. Katz, L., 1965. Unified treatment of a broad class of discrete probability distributions. In: Patil, G.P. (Ed.), Classical and Contagious Discrete Distributions. Statistical Publishing Society, Calcutta, India, pp. 175–185.
18. Kumar, V., Taneja, H.C., 2011. Some characterization results on generalized cumulative residual entropymeasure. Statistics & Probability Letters 81, 1072–1077.
19. Lerch, M., 1887. Note sur la function. Acta Mathematica 11, 19–24.
20. Lin, X., Willmot, G.E., 1999. Analysis of defective renewal equation arising in ruin theory. Insur- ance: Mathematics and Economics 25, 63–84.
21. Lin, X., Willmot, G.E., 2000. The moments of the time of ruin, the surplus before ruin and the deficit at ruin. Insurance: Mathematics and Economics 27, 19–44.
22. Magnus, W., Oberhettinger, F., Soni, R.P., 1966. Formulas and Theorems for the Special Functions of Mathematical Physics. Springer-Verlag, New York.
23. Mandelbrot, B., 1959. A note on a class of skew distribution functions: analysis and critique of a paper by H.A. Simmon. Information and Control 2, 90–99.
24. application to exponential and Weibull families. Biometrika 84, 641–652
25. Marshall, A.W., Olkin, I., Arnold, B.C., 2011. Inequalities: Theory of Majorization and Its Appli- cations, second edition. Springer-Verlag, New York
26. Nair, N.U., Nair, K.R.M., 1990. Characterizations of a bivariate geometric distribution. Statistica 50, 247–253.