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# Modeling of Abnormal Blood flow through Arteries: Presence of Obstacle with Slip condition

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**ABSTRACT**

In this precise mathematical calculation we have got modeling of abnormal blood flow. This fluid model is used to get axial velocity of blood with slip condition in the presence of obstacle i.e. stenosis. Abnormality results of Non-Newtonian condition of blood are displayed graphically for different flow properties with different parameters like length of contraction, viscosity of blood, stenosis length and height etc. The significance of present study may be useful to identify same cardio vascular deceases.

## 1. Introduction

Stenosis may become a reason for critical movement conditions concerned to artery collapse, plaque rupture which goes directly to stroke as well as heart attack. Stroke as well as heart problem, which occurring from severe stenosis, are the common causes of death in the societies. Stenosis increases in arteries which forces the human body to augment the blood pressure to maintain the required blood supply. Both the elevated pressure as well as the lessening of blood vessel is responsible for high movement velocity, high shear stress as well as low shear stress. An improved mathematical calculation in this physiological development is of great significance to early identification, prevention as well as curing stenosis concerned diseases.

In this paper we have put introduction, Review of literature, material and methods, results, conclusions and references on I, II, III, IV and V number respectively.

## 2. Review of Literature

The flow of liquid through a various stenosis arteries under the effect of external applied magnetic area is examined by Bali and Awasthi [2012]. Chakravarty et al. [1996] have dealt with the consequence of a solitary cycle of human body accelerations on unsteady run behavior of blood past a time-dependent artery stenosis. Chakravarty et al. [2009] stated that atherosclerotic tubes deal with precise models that symbolize non-Newtonian motion of liquid through a narrow tube in the existence of magnetic area. Das and Saha [2009] stated that a precise model for pulsatile motion of blood through a stenosis thick medium with periodic human body acceleration under the effect of a uniform magnetic area has been progressed by judged the blood to be Newtonian along incompressible motion. The unsteady movement of non-Newtonian liquid through merged stenosis is examined by Ellahi et al. [2013]. Jamalabadi et al. [2016] paid attention on transient simulation of blood motion through a narrowed stenosis artery enclosed by solenoid under the existence of heat transmit. Misra and Shit [2006] have done a study on Blood flow through arteries in a pathological state: A theoretical Study. A micro polar replica for axe-symmetric blood motion through tapered artery is examined by Mekheimer and Kot [2008]. The consequences of bell size stenosis on streamline contour, layer pressure drop and increase in shear stress for the reduce of the syndrome, atherosclerosis was studied numerically by Mandal et al. [2010]. Mathur and Jain explained [2013] a mathematical replica for studying the non -Newtonian motion of blood through a stenosis artery section. Mohan and Raghav [2017] explained about blood movements in arterioles. Sahu et al. [2010] dealt with the paired precisely model of blood motion for a narrowed artery in the existence of axe variable and peripheral wall. The objective of this study was to develop a precise model for examining the motion of blood through a stenosis artery area. Herschel–Bulkley model has been considered to represent the properties of blood. The difficulty was examined by a combined employ of precise and mathematical techniques. To estimate the consequences of the stenosis size, geometry has been followed such that the axe size of the narrowed pipe can be changed cleanly just by fluctuating a parameter. Singh and Rajbala [2010] investigated two-dimensional synthesis of blood motion with variable viscosity through stenosis in the existence of transverse magnetic area in the thick medium. Sharma and Raghav [2018] have a done a detailed study about hemodynamics flow and its effects in arteries.

## 3. Material and Methods

A steady laminar entirely developed one-dimensional movement of blood following the Herschel–Bulkley mathematical model through a diseased artery. The radius of the blood vessel depends upon stenosis and a mathematical geometry of bell shaped stenosis is

$$R = R_0\left[1 - \frac{\psi}{R_0} e^{\frac{-m^2\alpha^2 z^2}{R_0^{\,2}}}\right] \tag{1}$$

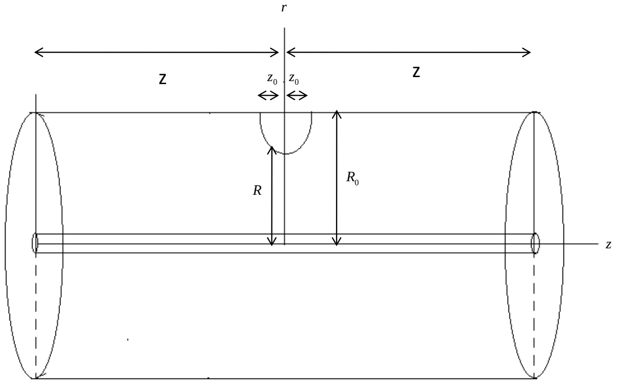



**Figure 1: Geometry of Stenosis in an artery**

Where $R_0$ is the radius of an artery, R(z) is the radius of an artery at stenosed segment , Z is the half magnitude of artery whereas ($z_0$) is the half magnitude of stenosed artery, n is fluid index, ψ is stenosed height, m is a parametric constant, r and z are radial and axial coordinate respectively, α is characterizes the concerned magnitude of the constriction, described as the ratio of radius to half magnitude of stenosis, i.e.,

$$\alpha=\frac{R_0}{z_0} \tag{2}$$

Assuming the stenosis precise geometry to be

$$\frac{\mathrm{R(z)}}{R_0}=\left[1-\upsilon e^{-\beta z^2}\right] \tag{3}$$

With $\upsilon=\frac{\psi}{R_0}$ and $\beta=\frac{m^2\alpha^2}{R_0^{\ 2}}$

If blood is a power law fluid then constitutive equation is

$$\tau=\mu e^n \quad \Rightarrow\left(\frac{\tau}{\mu}\right)^{\frac{1}{n}}=e=\left(\frac{\frac{1}{2}\mathrm{Pr}}{\mu}\right)^{\frac{1}{n}}=-\frac{du}{dr}$$

$$u=-\left(\frac{\mathrm{P}}{2\mu}\right)^{\frac{1}{n}}\frac{n}{n+1}r^{\frac{1}{n}+1}+C \tag{4}$$

Subject to the boundary conditions

$\tau$ is finite at r = 0 (Regularity Condition) (5)

$u=u_s$ at r = R(z) (Slip Condition) (6)

$$u_s=-\left(\frac{\mathrm{P}}{2\mu}\right)^{\frac{1}{n}}\frac{n}{n+1}R^{\frac{1}{n}+1}+C \tag{7}$$

$$u=\left(\frac{\mathrm{P}}{2\mu}\right)^{\frac{1}{n}}\frac{n}{n+1}\left(R^{\frac{1}{n}+1}-r^{\frac{1}{n}+1}\right)+u_s \tag{8}$$

Flow rate (Q) = $\int_0^R u2\pi rdr\Rightarrow 2\pi\int_0^R (u)rdr$

$$=2\pi\left(\frac{\mathrm{P}}{2\mu}\right)^{\frac{1}{n}}\frac{n}{n+1}\int_0^R\left(R^{\frac{1}{n}+1}.r-r^{\frac{1}{n}+2}\right)dr+2\pi u_s\int_0^R rdr$$

$$Q=2\pi\left(\frac{\mathrm{P}}{2\mu}\right)^{\frac{1}{n}}\frac{n}{n+1}R^{\frac{1}{n}+3}\left[\frac{1+n}{2(1+3\mathrm{n})}\right]+\pi u_s R^2$$

$$Q-\pi u_s R^2=\pi\left(\frac{\mathrm{P}}{2\mu}\right)^{\frac{1}{n}}\frac{n}{n+1}R^{\frac{1}{n}+3}\left[\frac{1+n}{(1+3\mathrm{n})}\right] \tag{9}$$

$$P=2\mu\left[\frac{\left(Q-\pi u_s R^2\right)}{\pi}\frac{(1+3\mathrm{n})}{n\left(R^{\frac{1}{n}+3}\right)}\right]^n \tag{10}$$

Pressure drop across the length of the stenosis

$$\Delta P=\frac{2\mu}{\left(R_0^{1+3n}\right)}\left[\frac{\left(Q-\pi u_s R^2\right)}{\pi}\frac{(1+3\mathrm{n})}{n}\right]^n\int_{-z_0}^{z_0}\frac{dz}{\left[1-\upsilon e^{-\beta z^2}\right]^{1+3n}} \tag{11}$$

If there is no stenosis, then

$$\left(\Delta P\right)_p=\frac{4\mu}{\left(R_0^{1+3n}\right)}\left[\frac{\left(Q-\pi u_s R^2\right)}{\pi}\frac{(1+3\mathrm{n})}{n}\right]^n z_0 \tag{12}$$

$$K=\frac{\Delta P}{\left(\Delta P\right)_p}=\frac{1}{2z_0}\int_{-z_0}^{z_0}\frac{dz}{\left[1-\upsilon e^{-\beta z^2}\right]^{1+3n}} \tag{13}$$

If total length of stenosed artery is 2L then pressure drop along the total length of artery

$$P=\frac{2\mu}{\left(R_0^{1+3n}\right)}\left[\frac{\left(Q-\pi u_s R^2\right)}{\pi}\frac{(1+3\mathrm{n})}{n}\right]^n\left(2L-2z_0\right)$$

$$+\frac{2\mu}{\left(R_0^{1+3n}\right)}\left[\frac{\left(Q-\pi u_s R^2\right)}{\pi}\frac{(1+3\mathrm{n})}{n}\right]^n\int_{-z_0}^{z_0}\frac{dz}{\left[1-\upsilon e^{-\beta z^2}\right]^{1+3n}} \tag{14}$$

In the absence of stenosis

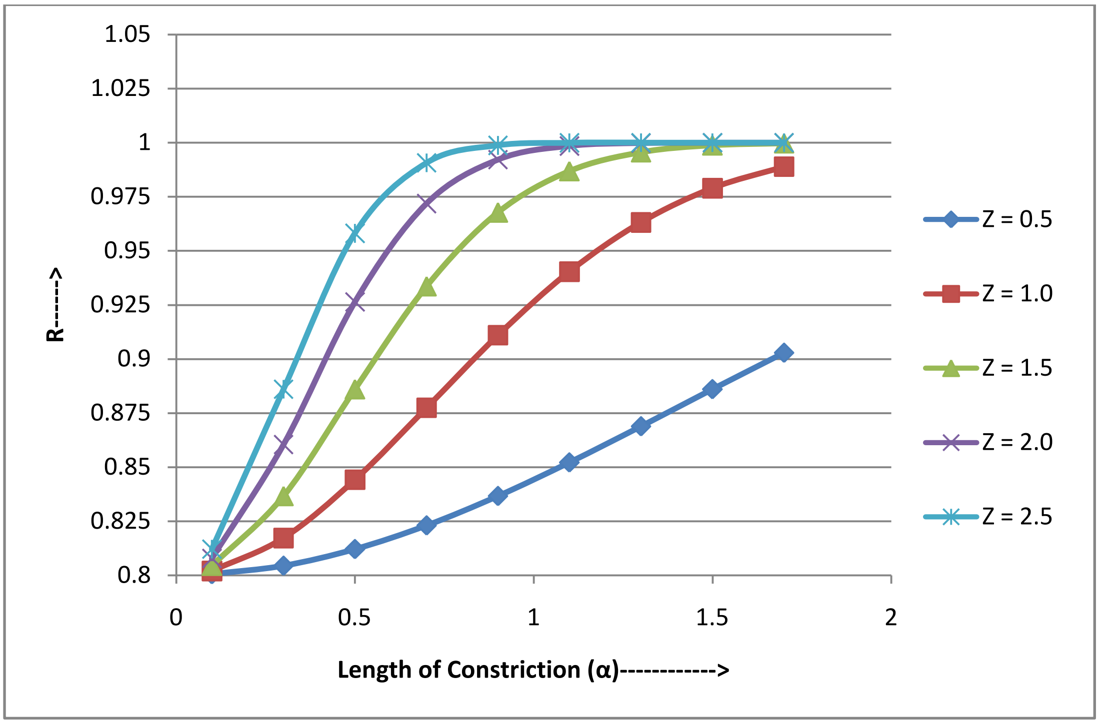
$$\left(\Delta P\right)_p=\frac{4\mu}{\left(R_0^{1+3n}\right)}\left[\frac{\left(Q-\pi u_s R^2\right)}{\pi}\frac{(1+3\mathrm{n})}{n}\right]^n L \tag{15}$$

$$K=\frac{\Delta P}{\left(\Delta P\right)_p}=1-\frac{z_0}{L}+\frac{1}{2L}\int_{-z_0}^{z_0}\frac{dz}{\left[1-\upsilon e^{-\beta z^2}\right]^{1+3n}} \tag{16}$$

## 4. Results

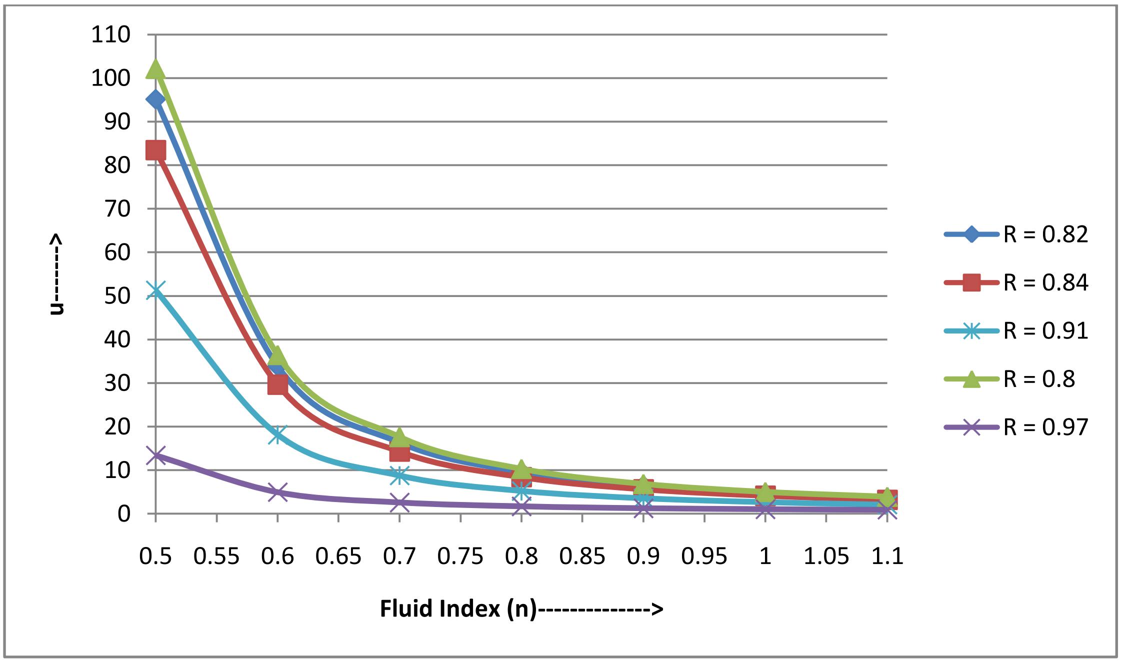
The objective of this study is to discuss the result of many parameters on the physiologically significant flow quantities like viscosity coefficient, stenosis length, height, length of constriction, fluid index, artery length etc.

The ranges of parameters are shown in graph 1 to graph 4.




**Graph 1: Variation of radius of artery at stenosed segment**
**Ψ = 0.2, m = 1, R0 = 1**

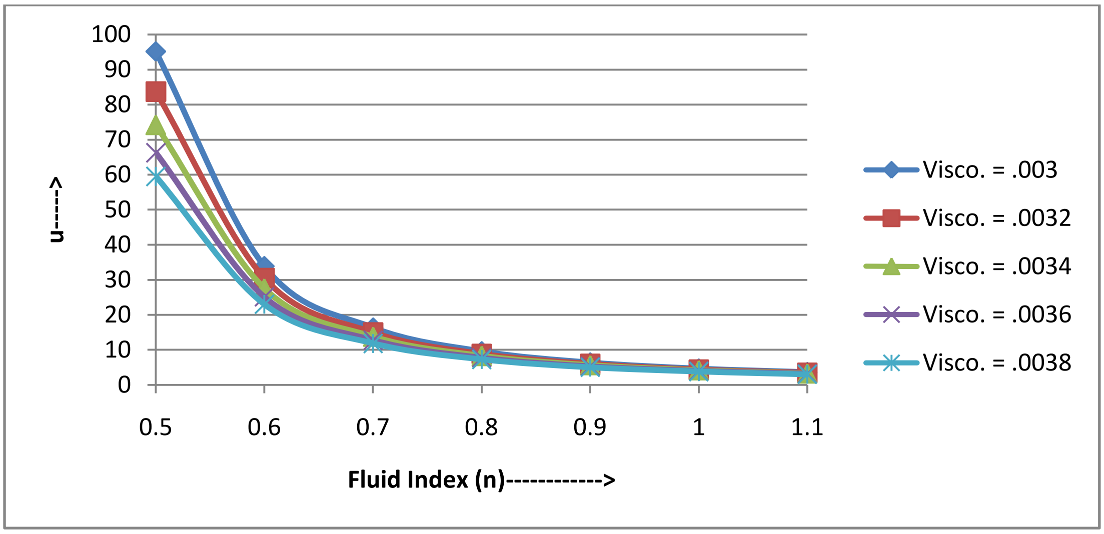
R shows a strong dependence on the values of α and Z. the value of R equal to 1 is an indication of the absence of stenosis. This can be seen in graph (1) where for a given value of Z, the R will be unified beyond a certain threshold value of α. However, one may notice a similar behavior for lower values of α but Z should be higher for an example, for α = 0.9 one should have $Z \geq 3.0$ beyond which one can obtain R = 1. Some parameters are taken constantly as shown in the graph (1).




**Graph 2: Variation of Axial Velocity at different Values of R and Fluid Index (n)**
**P = 0.15, $u_s$ = 0.5, $\mu$ = 0.003**

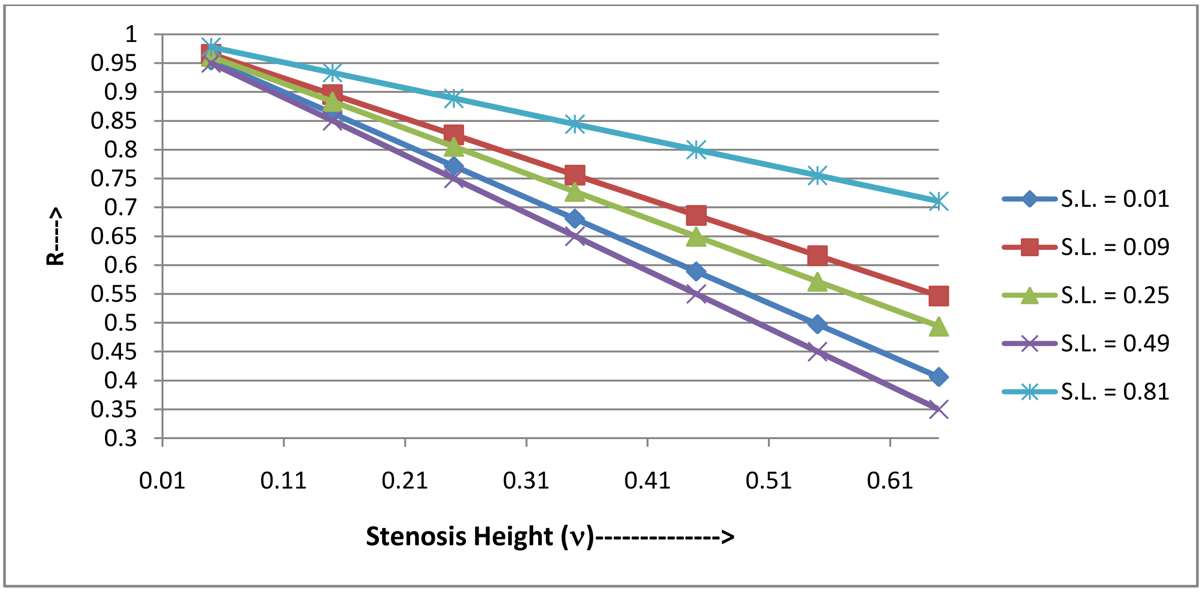
 

It may be seen from the graph (2) that axial velocity shows a strong dependence on R and n with a gradual increase in the values of R and Fluid index. The axial velocity drops down abruptly. This is medically not suitable for mostly arteries. As we know standard velocity for subclavian and brachial arteries and common and internal carotid arteries lies among 80 to 120 while anterior and middle cerebral arteries it lies among 30 to 70. It must be lies between standard values so that arteries work normally. Lowering the velocity below normal value creates a fatal situation for the cardiovascular system. Some parameters are taken constantly as shown in the graph (2).




**Graph 3: Variation of Axial Velocity at different viscosities**
**P = 0.15, $u_s$ = 0.5, R = 0.82**

It may be again seen from the graph (3) that axial velocity shows a strong dependence on viscosity coefficient ($\mu$) and fluid index (n) with a gradual increase in the values of R and Fluid index. The axial velocity drops down abruptly. This is medically not suitable for mostly arteries. As we know standard velocity for subclavian and brachial arteries and common and internal carotid arteries have exist among 80 to 120 c.m./second while anterior and middle cerebral arteries it lies between 30 to 70 c.m./second. It must be lies between standard values so that arteries work normally. Lowering the velocity below normal value creates a fatal situation for the cardiovascular system. Some parameters are taken constantly as shown in the graph (3).




**Graph 4: Variation of the radius of an artery at stenosed segment at different stenosis length and height**
**M =1, $R_0$ = 1, ψ = 0.2**

It may be seen from the graph (4) that the variation of the radius of an artery at stenosed segment relies on the values of stenosis length and height. The mathematical values of stenosis length and height are directly proportional to each other and their combination is inversely proportional to the radius of an artery at stenosed segment. Some parameters are taken constantly as shown in the graph (4).

## 5. Conclusion

Our study reveals that which numerical value of the parameter is ideal to prevent the size of stenosis. Further to remove that stenosis how numerical value of the certain parameter can be fixed. This study also tells that a radius of the stenosed artery affects to the axial velocity. Axial velocity also affects when viscosity coefficient change. This study also informs to us that radius of the stenosed artery is affected in the variation of stenosis length and height. Our study provides the influence of the various parameters in non- Newtonian blood flow. We found that the stenosis length, viscosity coefficient, stenosis height, length of constriction etc. The movement quantities indicate where the stenosis is formed. Thus our study tells the potential to explore the reasons for and development of arterial serious diseases like atherosclerosis and cardiovascular.

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