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Finding Shortest Path in a Network Using Trapezoidal Intuitionistic Fuzzy Number

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ABSTRACT

In this paper, we present an algorithm to find the shortest path from a source node to a destination node on a network using α-cut and Euclidean distance. We also solved a numerical example in which each edge assigns activity duration as trapezoidal intuitionistic fuzzy numbers.

1. Introduction

In a Network theory, one of the most common problems is to find the shortest path in a given network. The main objective is to find a path with minimum distance. As an extension of Lotfi Zadeh's notion set, Intuitionistic fuzzy sets have been introduced by Krassimir Atanassov(1983) which itself extends the classical notion of a set.

This problem is circumvented with the model based on the paper Fuzzy shortest path presented by P.Sandhiya [1]. The Fuzzy shortest path problem has a wide range of applications in different areas such as communication, route, transportation etc., many researches have focused on Fuzzy shortest problem on a network. In section 1 provides some elementary concepts need for this paper. In section 2 we proposed the algorithm for finding the shortest path in a Trapezoidal Intuitionistic fuzzy network problem. In section 3 we exemplify a numerical example which is presented in a paper [3]. In section 4 some conclusions are discussed.

1. PRELIMINARIES

DEFINITION 1.1: FUZZY SET

A fuzzy set A of a universal set X is defined by its membership function $\mu_A: X \to [0,1]$which assigns a real number$\mu_A(x)$ in the interval [0,1] to each element xϵX, where the value of $\mu_A(x)$ at x shows the grade of membership of x in A.

DEFINITION 1.2: INTUITIONISTIC FUZZY SET

An Intuitionistic fuzzy set A of a universal set X is given by A=$\{(x, \mu_A(x),\gamma_A(x))/x\epsilon X\}$ where the function $\mu_A(x)$: X → [0,1] and $\gamma_A(x)$: X → [0,1] determine the degree of membership and non-membership of the element xϵX respectively and for every xϵX, $0 \le \mu_A(x) + \gamma_A(x) \le 1$.

DEFINITION 1.3: INTUITIONISTIC FUZZY NUMBER

Let A = $\{(x, \mu_A(x),\gamma_A(x))/x\epsilon X\}$ be an intuitionistic fuzzy set, then we say $(\mu_A(x),\gamma_A(x))$ be an intuitionistic fuzzy number. For our convenience, letA=(<p, q><w, x>) be an intuitionistic fuzzy number where <p, q>$\in F(I) < w, x > \in F(\mathrm{I}). \mathrm{I} = [0,1]$ $0 \le c + n \le 1$.

DEFINITION 1.4: TRAPEZOIDAL FUZZY NUMBER

Let A = (p, q, r, s) be a trapezoidal fuzzy number whose membership function is given by

$$\mu_A(x) = \begin{cases} 0 & x < p \\ \frac{x-p}{q-p} & p \le x \le q \\ 1 & q \le x \le r \\ \frac{s-x}{s-r} & r \le x \le s \\ 0 & x > s \end{cases}$$

DEFINITION 1.5: TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBER

A Trapezoidal Intuitionistic fuzzy number A is denoted by $\{(\mu_A(x),\gamma_A(x))/x\epsilon X\}$ where $\mu_A(x)$ and $\gamma_A(x)$ are trapezoidal fuzzy number. So a trapezoidal intuitionistic fuzzy number A = (<p, q, r, s><v, w, y, z>) whose membership and non-membership function is given by

$$\mu_A(x) = \begin{cases} 0 & x < p \\ \frac{x-p}{q-p} & p \le x \le q \\ 1 & q \le x \le r \\ \frac{s-x}{s-r} & r \le x \le s \\ 0 & x > s \end{cases}$$

and

$$\gamma_A(x) = \begin{cases} 0 & x < v \\ \frac{w-x}{w-v} & v \le x \le w \\ 1 & w \le x \le y \\ \frac{x-y}{z-y} & y \le x \le z \\ 0 & x > z \end{cases}$$

DEFINITION 1.6: α-CUT FOR TRAPEZOIDAL INTUITIONISTICFUZZY NUMBER

Let A be the fuzzy set in X and any real number αϵ[0,1], it is denoted by α_A. then the α-CUTfor membership and non-membership function are

$$\mu_A(x) = \begin{cases} \frac{x-p}{q-p} & p \le x \le q \\ \frac{s-x}{s-r} & r \le x \le s \end{cases}$$

$$\gamma_A(x) = \begin{cases} \frac{w-x}{w-v} & v \le x \le w \\ \frac{x-y}{z-y} & y \le x \le z \end{cases}$$

Then the α-cut of A is α = $\frac{x-p}{q-p}$ and α = $\frac{s-x}{s-r}$ for membership function; α =$\frac{w-x}{w-v}$ and α =$\frac{x-y}{z-y}$ for non-membership function.

Expressing in terms of α, x = (q-p) α+p and x = s-(s-r) α for membership; x = w-(w-v) α and x = y + (z-y) α for non-membership function. Therefore, the α-cut of A is

α_A = (<(q- p) α +p, s-(s-r)α>< w-(w-v) α, y + (z-y) α>).

DEFINITION 1.7: OPERATION ON TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBER

Let$\tilde{A}$ = (<p, q, r, s><w, x, y, z>) and $\tilde{B}$ = (<p', q', r', s'><w', x', y', z'>)

Then ADDITION: $\tilde{A} \oplus \tilde{B}$ = (p + p', q + q', r + r', s +s′) (w + w', x + x', y + y', z + z')

DEFINITION 1.8: MINIMUM VALUE (MV) FOR α-CUTS

Let α_A = (<a,b><c, d>) and α_B= (<p, q><r, s>), then the minimum value is given by

MV = min (α_A,α_B)

$$= \left[\left(\min\left(\frac{a+b}{2}, \frac{p+q}{2}\right)\right)\left(\min\left(\frac{c+d}{2}, \frac{r+s}{2}\right)\right)\right]$$

DEFINITION 1.9: EUCLIDEAN DISTANCE

Let A = (<e, f><g, h>) and B = (<t, u><v, w>), then the Euclidean distance is given by

$$D = \left(\sqrt{(e-t)^2 + (f-u)^2}\right)\left(\sqrt{(g-v)^2 + (h-w)^2}\right)$$

2. ALGORITHM FOR FINDING SHORTEST PATH

STEP 1: FINDING POSSIBLE PATH

- Find all the possible paths from a source node to a destination node.
- Assign the number of possible paths in an acyclic network.

STEP2: COMPUTATION OF LENGTH OF PATHS

- Set α value between [0,1]
- Find the α-cut for every edge.
- By adding the α-cuts, find the length of all the possible paths.

STEP 3: COMPARISON OF PATHS

- Assume $L_{min} = L_1$
- Let i = 2 to n

MV = min (L_{min}, L_i)

$D_1 = D\,(MV, L_{min})$

$D_2 = D(MV, L_i)$

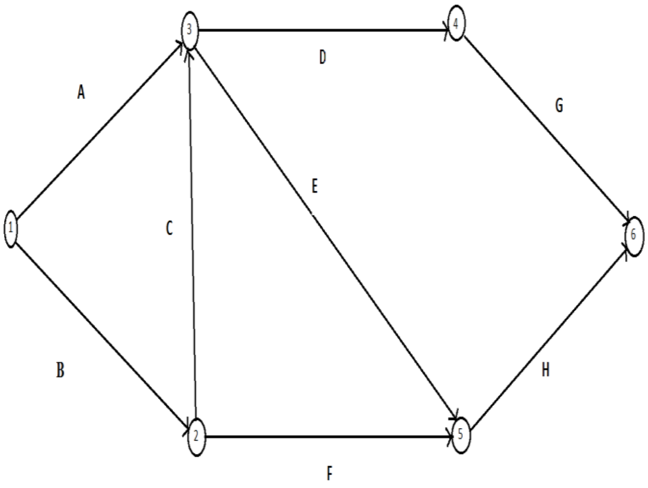
If $D_1 < D_2$ then $L_{min} = L_{min}$

Otherwise $L_{min} = L_i$

STEP 4: The shortest path is the corresponding path of L_{min}.

3. Numerical Example

Example: Consider a fuzzy acyclic network whose edge weights are taken as the trapezoidal intuitionistic fuzzy numbers.



The fuzzy arc lengths are:

A(1-3) = (<52, 62, 65, 70><47, 62, 65, 74>)

B (1-2) = (<10, 20, 20, 30><8, 20, 20, 35>)

C (2-3) = (<35, 38, 40, 45><30, 38, 40, 50>)

D (3-4) = (<10, 13, 17, 20><9, 13, 17, 25>)

E (3-5) = (<8, 9, 9, 10><6, 9, 9, 15>)

F (2-5) = (<52, 55, 60, 75><46, 55, 60, 70>)

G (4-6) = (<70, 75, 85, 97><64, 75, 85, 100>)

H (5-6) = (<50, 70, 80, 100><47, 70, 80, 105>)

STEP 1:FINDING ALL POSSIBLE PATHS

- Possible paths are

P_1 : A → D → G

P_2 : B→ F → H

P_3 : B → C → E → H

P_4 : A→ E → H

P_5 B→ C → D → G

- The number of possible paths is5. □N = 5

STEP 2: COMPUTATION OF LENGTH OF PATHS

- Set α value between 0 and 1.

Let α = 0.5

- **α –** cuts for every edge

α_A = (<57, 67.5><54.5, 69.5>)

α_B= (<15, 25><14, 27.5>)

α_C = (<36.5, 42.5><34, 45>)

α_D =(<11.8, 18.5><11, 21>)

α_E= (<8.5, 9.5><7.5, 12>)

α_F =(<53.5, 67.5><50.5, 65>)

α_G = (<72.5, 91><69.5, 92.5>)

α_H = (<60, 90><58.5, 92.5>)

- Length of the possible path

L_1 = (<141.3, 177><135, 183>)

L_2 = (<128.5, 182.5><123, 185>)

L_3 = (<120, 167><114, 177>)

L_4 = (<125.5, 167><120.5, 174>)

L_5 = (<135.8, 177><128.5, 186>)

STEP 3: COMPARISON OF PATHS

- Assume $L_{min} = L_1$ = (<141.3, 177><135, 183>)
- For i=2

MV = min (L_{min}, L_2)
= (<128.5, 182.5><123, 185>)
D_1= D (MV,L_{min})
= (<13.9316><12.1655>)
D_2= D (MV,L_2)
= (<0><0>)
Here$D_2 < D_1$,
□$L_{min} = L_2$ = (<128.5, 182.5><123, 185>)

- For i=3

MV = min (L_{min}, L_3)
= (<120, 167><114, 177>)
D_1= D (MV,L_{min})
= (<17.6777><12.0416>)
D_2= D (MV,L_3)
= (<0><0>)
Here$D_2 < D_1$,
□$L_{min} = L_3$ = (<120, 167><114, 177>)

- For i=4

MV = min (L_{min}, L_4)
= (<120, 167><114, 177>)
D_1= D (MV,L_{min})
= (<0><0>)
D_2= D (MV,L_4)
= (<5.5><7.1589>)
Here$D_2 > D_1$,
□$L_{min} = L_4$ = (<120, 167><114, 177>)

- For i=5

MV = min (L_{min}, L_5)
= (<120, 167><114, 177>)
D_1= D (MV,L_{min})
= (<0><0>)
D_2= D (MV,L_5)
= (<18.6987><17.066>)
Here$D_2 > D_1$,
□$L_{min} = L_5$ = (<120, 167><114, 177>)

STEP 4:

The optimized shortest path is B→ C → E → H and the distance is (<103, 137, 149,185><91, 137, 149, 205>).

4. Conclusion

In this paper, we presented an algorithm with the numerical example for finding the shortest path using α-cut set and Euclidean distance in which each edge is assigned to a Trapezoidal Intuitionistic Fuzzy Number. The fuzzy shortest path problem is solved for each node from the source node to a destination node and hence the optimized critical path is obtained.

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