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Analysis of Different Parameters of Finite Queue Single Server Queuing Model by Using Waiting Time Distribution

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ABSTRACT
The aim of this paper is to analyze the performance measures of various parameters of finite queue single server model by using the relation of waiting time of customer in the queue. By using binomial distribution waiting time of customer in the queue is derived first and using it to find the other related parameters.

1. Introduction

The model we discuss here is a single server finite queue model. Capacity of the system is finite, only finite numbers of customers are allowed in the system. Arrival rate of customers and service rate of server follows Binomial distribution and inter-arrival time, service time follows Uniform distribution. By using Binomial distribution for the arrival rate of customers and service rate of server, formula to find waiting time of a customer in the queue is derived. By using this result various others effective measures are calculated and analyzed. Application of Binomial distribution gives the time assurance of finite number of arrivals and time required to serve the finite number of customers. The performance measures which are discuss here to analyze the system are waiting time of a customer in the queue, waiting time of a customer in the system, number of customers in the queue, number of customers in the system, **w**aiting time of a customer in the queue for busy system, number of customers served per busy period and probability of customer has to wait for more than a particular time.

2. Methodology

Let capacity of system be 'M' and 'α'be the average arrival rate of customers. 'β' be the average service rate of customer. Model works under the queue discipline (FCFS).

Utilization factor= server is busy for service is $\rho = \dfrac{\alpha}{\beta}$. (1)

Probability of 'x' customers arrived in time't' is $P_x(t)={}^MC_x\left(\dfrac{\alpha t}{M}\right)^x\left(1-\dfrac{\alpha t}{M}\right)^{M-x}, t<\dfrac{M}{\alpha}$ (2)

From this relation it is gives an assurance that the time required for the arrival of 'M' number of customers is $o \le t < \dfrac{M}{\alpha}$.

Probability of 'x' customers served by the server in time 't' is

$$P_x(t)={}^MC_x\left(\frac{\beta t}{M}\right)^x\left(1-\frac{\beta t}{M}\right)^{M-x},\ t<\frac{M}{\beta} \quad (3)$$

From this relation it is clear that the service time required for 'M' customers is $o \le t < \dfrac{M}{\beta}$.

Probability of 'x' customer in the system is

$$P_x=\left(\frac{\alpha}{\beta}\right)^x\left(\frac{1-\dfrac{\alpha}{\beta}}{1-\left(\dfrac{\alpha}{\beta}\right)^{M+1}}\right)=(\rho)^x\left(\frac{1-\rho}{1-\rho^{M+1}}\right), x=0,1,2,.......M\ , \rho\ \neq 1, \alpha\neq\beta \quad (4)$$

$$= \frac{1}{M+1}, \ \rho \ = 1, \alpha = \beta \qquad (5)$$

2.1 Little's Formula:

$$N_s = \alpha' T_s \ N_q = \alpha' T_q \qquad (6)$$

Where $\alpha' = \alpha_{eff} = \alpha(1 - P_M)$ which is effective average arrival rate

Where N_s is expected number of customers in the system, N_q is expected number of customers in the queue, α' is effective average arrival rate of customer , T_s is the waiting time of customer in the system and T_q is the waiting time of customer in the queue.

3. Waiting Time Distribution of a Customer in the Queue

When the customer is arriving to avail the service facility has to wait in the queue because some other customers are already in the queue. As the queue discipline is (FCFS), unless and until the new arriving customer get the service after the completion of service of all the customers those who are already in the queue. In steady state condition the waiting time distribution of each customer is same and a continuous random variable. Let $'T'$ be the time required by the server to serve all the customers in the system.

Let $F_T(t)$ be the probability distribution function of $'T'$

Where, $$F_T(0) = P_0 = \frac{1-\rho}{1-\rho^{M+1}} \qquad (7)$$

If a customer is arriving for service and there are already $x \geq 1$ customers present in the system then the arriving customer will get service after the completion of service of all the customers in the system. As the capacity of system is M , arriving customer is getting service after the service completion of $(M-1)$ customers.

Let $f_x(t)$ be the probability density function of $1 \leq x \leq (M-1)$ customers.

$$\therefore \ f_x(t) = \sum_{x=1}^{M-1} P_x [\Pr ob\{(x-1) \ customers \ got \ service \ at \ time \ 't'\}$$

$$\times \Pr ob\{one \ customer \ is \ under \ service \ during \ time \ \omega t\}]dt$$

(Where ωt is very small)

Since β is the average service rate of a server

$\therefore$ Probability of a customer under service during time ωt = $\beta \omega t$

$$\therefore f_x(t) = \sum_{x=1}^{M-1} \rho^x \frac{1-\rho}{1-\rho^{M+1}} \left[{}^{M-1}C_{x-1} \left(\frac{\beta t}{M-1} \right)^{x-1} \left(1 - \frac{\beta t}{M-1} \right)^{(M-1)-(x-1)} \right] \beta \omega t \qquad (8)$$

(From equations (3), & (4))

$$\therefore F_T(t) = P(T \leq t) = F_T(0) + \int_0^t \sum_{x=1}^{M-1} \rho^x \frac{1-\rho}{1-\rho^{M+1}} \left[{}^{M-1}C_{x-1} \left(\frac{\beta t}{M-1} \right)^{x-1} \left(1 - \frac{\beta t}{M-1} \right)^{(M-x)} \right] \beta \, dt$$

(From equations (8))

By diff w. r. to't' we get probability density function of waiting time distribution

$$F_T'(t) = 0 + \sum_{x=1}^{M-1} \rho^x \frac{1-\rho}{1-\rho^{M+1}} \left[{}^{M-1}C_{x-1} \left(\frac{\beta \ t}{M-1} \right)^{x-1} \left(1 - \frac{\beta \ t}{M-1} \right)^{(M-x)} \right] \beta$$

(Since from equation (7), $F_T(0)$ is independent of't')

$$\therefore F_T'(t) = \sum_{x=1}^{M-1} \rho^x \frac{1-\rho}{1-\rho^{M+1}} \left[{}^{M-1}C_{x-1} \left(\frac{\beta \ t}{M-1} \right)^{x-1} \left(1 - \frac{\beta \ t}{M-1} \right)^{(M-x)} \right] \beta \qquad (9)$$

Now, time required to serve $(M-1)$ number of customers is $0 \leq t < \frac{M-1}{\beta}$

Hence expected waiting time of a customer waiting in the queue is given by

$$T_q = \int_0^{M-1/\beta} t \times F_T'(t)dt$$

$$T_q = \int_0^{M-1/\beta} t \times \sum_{x=1}^{M-1} \rho^x \frac{1-\rho}{1-\rho^{M+1}} \left[{}^{M-1}C_{x-1} \left(\frac{\beta\, t}{M-1} \right)^{x-1} \left(1 - \frac{\beta\, t}{M-1} \right)^{(M-x)} \right] \beta dt$$

$$T_q = \beta \frac{1-\rho}{1-\rho^{M+1}} \sum_{x=1}^{M-1} \rho^x \, {}^{M-1}C_{x-1} \left(\frac{\beta}{M-1} \right)^{x-1} \int_0^{M-1/\beta} t^x \left(1 - \frac{\beta t}{M-1} \right)^{(M-x)} dt$$

To solve the integration use substitution

$$\frac{\beta t}{M-1} = y \Rightarrow t = \frac{M-1}{\beta} y \Rightarrow dt = \frac{M-1}{\beta} dy \ , \ 0 \le y \le 1$$

$$\therefore T_q = \beta \frac{1-\rho}{1-\rho^{M+1}} \sum_{x=1}^{M-1} \rho^x \, {}^{M-1}C_{x-1} \left(\frac{\beta}{M-1} \right)^{x-1} \int_0^1 \left(\frac{M-1}{\beta} y \right)^x (1-y)^{(M-x)} \frac{M-1}{\beta} dy$$

$$= \beta \frac{1-\rho}{1-\rho^{M+1}} \sum_{x=1}^{M-1} \rho^x \, {}^{M-1}C_{x-1} \left(\frac{\beta}{M-1} \right)^{x-1} \left(\frac{M-1}{\beta} \right)^{x+1} \int_0^1 y^x (1-y)^{(M-x)} dy$$

$$= \beta \frac{1-\rho}{1-\rho^{M+1}} \sum_{x=1}^{M-1} \rho^x \, {}^{M-1}C_{x-1} \left(\frac{M-1}{\beta} \right)^2 \beta'(x+1, M-x+1)$$

$$= \beta \frac{1-\rho}{1-\rho^{M+1}} \sum_{x=1}^{M-1} \rho^x \, {}^{M-1}C_{x-1} \left(\frac{M-1}{\beta} \right)^2 \frac{\Gamma(x+1)\Gamma(M-x+1)}{\Gamma(M+2)}$$

$$= \beta \frac{1-\rho}{1-\rho^{M+1}} \sum_{x=1}^{M-1} \rho^x \frac{(M-1)!}{(x-1)!(M-x)!} \left(\frac{M-1}{\beta} \right)^2 \frac{x!(M-x)!}{(M+1)!}$$

$$= \frac{\beta}{M(M+1)} \frac{1-\rho}{1-\rho^{M+1}} \left(\frac{M-1}{\beta} \right)^2 \sum_{x=1}^{M-1} \rho^x x$$

Hence expected waiting time of a customer in the queue is

$$T_q = \frac{\beta}{M(M+1)} \frac{1-\rho}{1-\rho^{M+1}} \left(\frac{M-1}{\beta} \right)^2 \sum_{x=1}^{M-1} \rho^x x \qquad (10)$$

By using equation (10) other remaining queuing parameters of the model can be calculated.

4. Performance Measures of Single Server Finite Queue Model:

4.1 Expected Waiting Time of a Customer in the System:

Expected waiting time of a customer in the system is given by

T_s = Expected waiting time of customer in the queue+ service time of customer

$$= T_q + \frac{1}{\beta} \qquad (11)$$

4.2 Expected Number of Customers in the Queue:

During the process of serving the customer by the server one customer is in service and remaining are in the queue. Therefore out of 'x' customers '$x-1$' are in the queue. So by using Little's formula (6) we have

Expected number of customers in the queue is given by

$N_q = \alpha' T_q$ Where $\alpha' = \alpha_{eff} = \alpha(1-P_M)$

$$\therefore N_q = \alpha \left(1 - \rho^M \frac{1-\rho}{1-\rho^{M+1}} \right) \times T_q \qquad (12)$$

4.3 Expected Number of Customer in the System:

It is the total number of customers in the queue and in the service mechanism which can be calculated by using Little's formula (6) we have

Expected number of customers in the system is given by

$N_s = \alpha' T_s$ Where $\alpha' = \alpha_{eff} = \alpha(1-P_M)$

$$\therefore N_s = \alpha\left(1-\rho^M \frac{1-\rho}{1-\rho^{M+1}}\right)\times T_s \qquad (13)$$

4.4 Expected Waiting Time of a Customer in the Queue for Busy System:

Expected waiting time of a customer in the queue for busy system is given by

$$T_b = \frac{T_q}{\text{Prob. of system being busy}}$$

$$= T_q \times \frac{1}{1-P_0} = T_q \times \frac{1-\rho^{M+1}}{\rho\ (1-\rho^M)} \quad \text{(From equation (7))} \qquad (14)$$

4.5 Expected Number of Customers Served Per Busy Period:

Expected number of a customer served per busy period is given by

$$N_b = \frac{N_s}{\text{Prob. of system being busy}}$$

$$= N_s \times \frac{1}{1-P_0} = N_s \times \frac{1-\rho^{M+1}}{\rho\ (1-\rho^M)} \quad \text{(From equation (7))} \qquad (15)$$

4.6 Probability of a Customer to Wait in the Queue for Particular Time:

Probability of a customer has to wait in the queue for more than time' a ' minutes where $0 \le t < \frac{M-1}{\beta}$ is

$$\text{P (Waiting} > a) = \int_a^{M-1/\beta} F_T'(t)dt$$

$$= \int_0^{M-1/\beta} \sum_{x=1}^{M-1} \rho^x \frac{1-\rho}{1-\rho^{M+1}} \left[{}^{M-1}C_{x-1} \left(\frac{\beta\, t}{M-1}\right)^{x-1} \left(1-\frac{\beta\, t}{M-1}\right)^{(M-x)} \right] \beta dt \quad \text{(From equation (9))}$$

$$= \frac{1-\rho}{1-\rho^{M+1}} \beta \int_0^{M-1/\beta} \sum_{x=1}^{M} \left[\rho^x\, {}^{M-1}C_{x-1} \left(\frac{\beta\, t}{M-1}\right)^{x-1} \left(1-\frac{\beta\, t}{M-1}\right)^{(M-x)} - \rho^M \left(\frac{\beta t}{M-1}\right)^{M-1} \right] dt$$

$$= \frac{1-\rho}{1-\rho^{M+1}} \beta \int_0^{M-1/\beta} \left[\rho\left(1-\frac{\beta t}{M-1}\right)^{M-1} + \rho^2 (M-1)\left(\frac{\beta t}{M-1}\right)\left(1-\frac{\beta t}{M-1}\right)^{M-2} \right.$$

$$\left. + \rho^3 \frac{(M-1)(M-2)}{2!}\left(\frac{\beta t}{M-1}\right)^2 \left(1-\frac{\beta t}{M-1}\right)^{M-3} + \ldots\ldots + \rho^M \left(\frac{\beta t}{M-1}\right)^{M-1} - \rho^M \left(\frac{\beta t}{M-1}\right)^{M-1} \right]$$

$$= \frac{1-\rho}{1-\rho^{M+1}} \beta\rho \int_a^{M-1/\beta} \left[\left(1-\frac{\beta\, t}{M-1}+\rho\frac{\beta\, t}{M-1}\right)^{M-1} - \left(\rho\frac{\beta\, t}{M-1}\right)^{M-1} \right] dt$$

$$= \frac{1-\rho}{1-\rho^{M+1}} \beta\ \rho \left[\frac{\left(1-\frac{\beta\, t}{M-1}+\frac{\rho\ \beta\ t}{M-1}\right)^M}{M\left(-\frac{\beta}{M-1}+\frac{\rho\ \beta}{M-1}\right)} - \frac{\rho^{M-1}\beta^{M-1}}{(M-1)^{M-1}} \frac{t^M}{M} \right]_a^{M-1/\beta}$$

$$= \frac{M-1}{M(1-\rho^{M+1})}\left[\rho\left(\left(1-\frac{\beta\, a}{M-1}(1-\rho\,)\right)^{M}-\rho^{M-1}\right)+\left(\frac{\rho\,\beta\, a}{M-1}\right)^{M}(1-\rho\,)\right] \qquad (16)$$

5. Conclusion

By using equation (10) and Little's formula (6) the other performance measures of the single server finite queue model can be calculated. The above mention parameters help to measure the effectiveness of the queuing model. The study of these parameters support to improve the system by applying some favourable changes in the model to increase the satisfaction level of the customers.

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