Application of Binomial Distribution and Uniform Distribution to Study the Finite Queue Multiple Server Queuing Model

[1]Damodhar F Shastrakar & [2]Sharad S Pokley

[1]Department of Mathematics, Smt Radhikatai Pandav College of Engineering, Nagpur (India)
[2]Department of Mathematics, KITS, Ramtek (India)

ARTICLE DETAILS

Article History
Published Online: 10 December 2018

Keywords
Average arrival rate, Average service rate, Average inter-arrival time, Average service time, Utilization factor

***Corresponding Author**
Email: dfshastrakar[at]yahoo.com

ABSTRACT

This paper is based on the analysis of various performance measures of queuing model having limited system capacity and multiple servers by using Binomial and Uniform probability distributions. By using these distributions operating characteristics of the said model is discuss in detail. The result of waiting time of customer in the queue is derived to study the remaining performance measures of queuing model.

1. Introduction

As the capacity of the system is finite, use of Binomial distribution follows arrival rate of the customers and service rate of server. The inter-arrival time of customers and service time of servers follows Uniform distribution. By using Binomial distribution the probability relation of number of customers in the system is derived. For the analysis of effective measures of the queuing system a relation of expected waiting time of a customer in the queue is derived by using Binomial distribution. Using this relation and Little's formula the other related performance measures are calculate and studied. Basic structure of finite queue multiple-server queuing model is given below Fig.1.

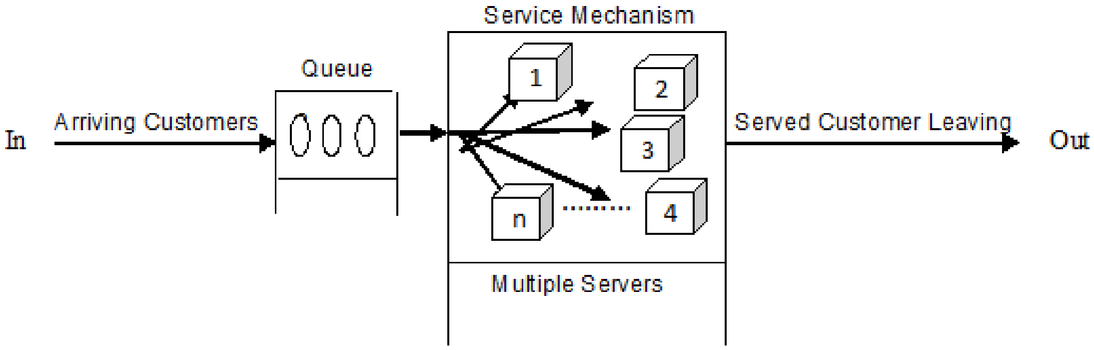

Fig.1 Basic Structure of Finite Queue Multiple Server Queuing Model

2. Methodology

Let the capacity of the system be 'M' and number of multiple servers be 'n' for service.
Let 'α 'be the average arrival rate of customers.

$\therefore$ Average inter-arrival time of customer = $\frac{1}{\alpha}$

Let 'β ' be the average service rate of each server.

$\therefore$ Average service time of a customer by each server = $\frac{1}{\beta}$

Let the number of customers in the system be $'x'$. Since there are 'n' multiple servers in the queuing model, the mean service rate of server is classified in to two possible cases.

Case-1: If the arriving customers are less than the number of servers $(x < n)$ then, average service rate is $= x\beta$.

Case-2: If the arriving customers exceeds or equal than the number of servers in the system $(x \geq n)$ then, mean service rate is $= n\beta$.

The utilization factor of servers can be calculated when all the servers are busy.

$\therefore$ Utilization factor $\rho = \dfrac{\alpha}{n\beta}$ (1)

The queuing model is working on queue discipline FCFS.

By using binomial distribution, probability of 'x' customers arrived in time't' is

$$P_x(t) = {}^{M}C_x \left(\frac{\alpha t}{M}\right)^x \left(1 - \frac{\alpha t}{M}\right)^{M-x}, t < \frac{M}{\alpha} \quad (2)$$

This relation gives an assurance that the time required for the arrival of 'M' number of customers is $0 \leq t < \dfrac{M}{\alpha}$.

For the probability of served customers

Case-1: For $(x < n)$

Probability of 'x' customers served by the servers less than 'n' in time't' is $P_x(t) = {}^{M}C_x \left(\dfrac{x\beta\, t}{M}\right)^x \left(1 - \dfrac{x\beta\, t}{M}\right)^{M-x}, \ t < \dfrac{M}{x\beta}$

(3)

This relation gives an assurance that the service time required to serve the customers which are less than the available servers is $0 \leq t < \dfrac{M}{x\beta}$.

Case-2: For $(x \geq n)$

Probability of 'x' customers served by the 'n' number of servers in time't' is $P_x(t) = {}^{M}C_x \left(\dfrac{n\beta\, t}{M}\right)^x \left(1 - \dfrac{n\beta\, t}{M}\right)^{M-x}, \ t < \dfrac{M}{n\beta}$

(4)

This relation gives an assurance that the service time required for 'M' customers is $0 \leq t < \dfrac{M}{n\beta}$.

3. Time Distribution of Finite Queue Multiple Server Queuing Model by Using Uniform Distribution

3.1 For arrival of customers:

By uniform distribution probability density function for continuous random variable't' is $f(t) = \dfrac{1}{b-a}$, $a \leq t \leq b$

$= 0$, *otherwise* (5) The cumulative distribution function of uniform distribution is $F(t) = \displaystyle\int_{-\infty}^{t} f(t)dt$

$$\therefore F(t) = \int_a^t \frac{1}{b-a} dt = \frac{1}{b-a}[t-a] \quad (6)$$

For arrival time distribution, average inter-arrival time of customer is $\dfrac{1}{\alpha}$ and mean of uniform distribution is $\dfrac{a+b}{2}$

$$\therefore \frac{1}{\alpha} = \frac{a+b}{2} \Rightarrow \alpha = \frac{2}{a+b} \Rightarrow b = \frac{2}{\alpha} - a$$

Here time starts from $t = 0$ which gives $a = 0 \Rightarrow b = \dfrac{2}{\alpha}$

$\therefore$ Probability density function is $f(t) = \dfrac{1}{\dfrac{2}{\alpha} - 0} = \dfrac{\alpha}{2}\ ,\ 0 \le t \le \dfrac{2}{\alpha}$ (7)

Hence probability distribution function is

$$F(t) = \frac{1}{b-a}[t-a] = \frac{1}{\dfrac{2}{\alpha}-0}[t-0] = \frac{\alpha t}{2} \quad , 0 \le t \le \frac{2}{\alpha} \ . \tag{8}$$

Which gives the probability of another customer arriving in the next coming time'tʼ if a customer already arrived.

The time limit gives an assurance of 100% arrival of another customer in the next coming time $\dfrac{2}{\alpha}$.

3.2 For service time distribution of customers:

Average service time a customer served by each server is $\dfrac{1}{\beta}$

$\therefore$ Probability density function is $f(t) = \dfrac{1}{\dfrac{2}{\beta} - 0} = \dfrac{\beta}{2}\ ,\ 0 \le t \le \dfrac{2}{\beta}$ (9)

And cumulative distribution function is

$$F(t) = \frac{\beta\, t}{2}\ ,\ 0 \le t \le \frac{2}{\beta} \tag{10}$$

Which gives the probability of another customer served in the next coming time'tʼ by each server if a customer already served.

The time limit gives an assurance of 100% served another customer in the next coming time $\dfrac{2}{\beta}$ by each server.

4. Relation of Probability for Number of Customers in the System

System will have probability of containing ‘x’ number of customers at time $(t + \omega t)$ is

$$\begin{aligned}
P_x(t+\omega t) &= P_x(t)\{\mathrm{Prob}(zero\ \ arrival\ in\ \ \omega t\ \&\ zero\ departure\ \ in\ \ \omega t)\} \\
&+ P_{x+1}(t)\{\mathrm{Prob}(zero\ \ arrival\ \ in\ \ \omega t\ \&\ one\ departure\ \ in\ \ \omega t)\} \\
&+ P_{x-1}(t)\{\mathrm{Prob}(one\ \ arrival\ in\ \omega t\ \&\ zero\ departure\ in\ \ \omega t)\}
\end{aligned}$$

(Where ωt is very small)

Case-1: For $(x < n)$

$$P_x(t+\omega t) = P_x(t)\left\{\left(1-\frac{\alpha\omega t}{M}\right)^M\left(1-\frac{x\beta\ \omega t}{M}\right)^M\right\} + P_{x+1}(t)\left\{\left(1-\frac{\alpha\omega t}{M}\right)^M M\left(\frac{(x+1)\beta\ \omega t}{M}\right)\left(1-\frac{(x+1)\beta\ \omega t}{M}\right)^{M-1}\right\}$$

$$+ P_{x-1}(t)\left\{M\left(\frac{\alpha\omega t}{M}\right)\left(1-\frac{\alpha\omega t}{M}\right)^{M-1}\left(1-\frac{(x-1)\beta\ \omega t}{M}\right)^M\right\}$$ (From equations (2) & (3))

$$\therefore P_x^{'}(t) = \alpha P_{x-1}(t) + \beta(x+1)\ \ P_{x+1}(t) - (\alpha + x\beta\ )P_x(t) \tag{11}$$

Case-2: For $(n \le x \le M)$

$$P_x(t+\omega t) = P_x(t)\left\{\left(1-\frac{\alpha\omega t}{M}\right)^M\left(1-\frac{n\beta\ \ \omega t}{M}\right)^M\right\}$$

$$+P_{x+1}(t)\left\{\left(1-\frac{\alpha\omega t}{M}\right)^{M} M\left(\frac{n\beta\omega t}{M}\right)\left(1-\frac{n\beta\omega t}{M}\right)^{M-1}\right\}$$

$$+P_{x-1}(t)\left\{M\left(\frac{\alpha\omega t}{M}\right)\left(1-\frac{\alpha\omega t}{M}\right)^{M-1}\left(1-\frac{n\beta\ \omega t}{M}\right)^{M}\right\}$$

(From equations (2) & (4))

On simplifying as mention in case-1, we get

$$\therefore P_x^{'}(t)=\alpha P_{x-1}(t)+\beta n\ P_{x+1}(t)-(\alpha+n\beta\ )P_x(t) \qquad (12)$$

At steady state condition system is independent of time. Hence under the steady state condition the recurrence relation of probability as

Probability of $'x'$ customer in the system is

$$P_x=\frac{1}{x!}\left(\frac{\alpha}{\beta}\right)^x P_0 \qquad ;(x<n) \qquad (13)$$

$$=\frac{1}{n!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x P_0 \qquad ;(n\le x\le M) \qquad (14)$$

Where (No customer for service) service stand idle

$$P_0=\left[\sum_{x=0}^{n-1}\frac{1}{x!}\left(\frac{\alpha}{\beta}\right)^x+\sum_{x=n}^{M}\frac{1}{n!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x\right]^{-1} \qquad (15)$$

5. Waiting Time Distribution of a Customer in the Queue:

As the queuing model is having ‘n’ number of multiple server, arriving customer has to wait in the queue only if the number of customers is greater than the number of servers. When the arriving customer are less than the number of server , at least one of the server stand idle and is available for the next arriving customer . Therefore to find the waiting time of customer in the queue only case-2 is applicable.

In steady state condition the waiting time distribution of each customer is same and a continuous random variable. Let $'T'$ be the time required by the servers to serve all the customers in the system.

Let $F_T(t)$ be the probability distribution function of $'T'$

Where,

$F_T(0)=$ Probability of no customer + probability of customers $(x<n)$

$$=1-P_0\sum_{x=n}^{M}\frac{1}{n!n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x \qquad (16)$$

If a customer is arriving for service and there are already $x\ge n$ customers present in the system then the arriving customer will get service after the completion of service of all the customers in the system. As the capacity of queue is M , arriving customer is allow to join the queue only if $(M-1)$ customers are there in the system and getting service after they all are served .

Let $f_x(t)$ be the probability density function of $n\le x\le(M-1)$ customers.

$$\therefore\ f_x(t)=\sum_{x=n}^{M-1}P_x[\Pr ob\{(x-n)\ customers\ got\ service\ at\ time\ 't'\}$$

$$\times\Pr ob\{n\ \ customer\ \ is\ \ under\ \ service\ during\ \ time\ \ \omega t\}]dt$$

(Where ωt is very small)

Probability of ‘n’ customers under service by ‘n’ servers during time ωt $=n\beta\omega t$

$$\therefore f_x(t)=\sum_{x=n}^{M-1}\frac{1}{n!}\frac{1}{n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x P_0\left[{}^{M-1}C_{x-n}\left(\frac{n\beta\ t}{M-1}\right)^{x-n}\left(1-\frac{n\beta\ t}{M-1}\right)^{(M-1)-(x-n)}\right]n\beta\omega t \qquad (17)$$

(From equations (4) & (14))

$$\therefore F_T(t) = P(T \le t) = F_T(0) + \int_0^t \sum_{x=n}^{M-1} \frac{1}{n!}\frac{1}{n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x P_0 \left[{}^{M-1}C_{x-n} \left(\frac{n\beta\, t}{M-1}\right)^{x-n} \left(1-\frac{n\beta\, t}{M-1}\right)^{(M-1)-(x-n)} \right] n\beta dt$$

(From equation (17))

By diff w. r. to 't' we get probability density function of waiting time distribution

$$F_T'(t) = 0 + \sum_{x=n}^{M-1} \frac{1}{n!}\frac{1}{n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x P_0 \left[{}^{M-1}C_{x-n} \left(\frac{n\beta\, t}{M-1}\right)^{x-n} \left(1-\frac{n\beta\, t}{M-1}\right)^{(M-1)-(x-n)} \right] n\beta$$

(Since from equation (16), $F_T(0)$ is independent of't')

$$= \frac{1}{(n-1)!} P_0 \beta \left[\sum_{x=n}^{M-1} \frac{1}{n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x {}^{M-1}C_{x-n} \left(\frac{n\beta\, t}{M-1}\right)^{x-n} \left(1-\frac{n\beta\, t}{M-1}\right)^{(M-1)-(x-n)} \right] \qquad (18)$$

Now, time required to serve $M-1$ number of customers is $0 \le t < \dfrac{M-1}{n\beta}$

Hence expected waiting time of a customer waiting in the queue is given by

$$T_q = \int_0^{M-1/n\beta} t \times F_T'(t) dt$$

$$= \int_0^{M-1/n\beta} t \times \frac{1}{(n-1)!} P_0 \beta \left[\sum_{x=n}^{M-1} \frac{1}{n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x {}^{M-1}C_{x-n} \left(\frac{n\beta\, t}{M-1}\right)^{x-n} \left(1-\frac{n\beta\, t}{M-1}\right)^{(M-1)-(x-n)} \right] dt$$

(From equation (18))

$$= \frac{1}{(n-1)!} P_0 \beta \left[\sum_{x=n}^{M-1} \frac{1}{n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x {}^{M-1}C_{x-n} \int_0^{M-1/n\beta} t \times \left(\frac{n\beta\, t}{M-1}\right)^{x-n} \left(1-\frac{n\beta\, t}{M-1}\right)^{(M-1)-(x-n)} dt \right]$$

$$= \frac{1}{(n-1)!} P_0 \beta \left[\sum_{x=n}^{M-1} \frac{1}{n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x \left(\frac{n\beta}{M-1}\right)^{x-n} {}^{M-1}C_{x-n} \int_0^{M-1/n\beta} t^{x-n+1} \left(1-\frac{n\beta t}{M-1}\right)^{(M-1)-(x-n)} dt \right]$$

By using substitution $\dfrac{n\beta t}{M-1} = y \Rightarrow t = \dfrac{(M-1)y}{n\beta}, dt = \dfrac{(M-1)}{n\beta} dy, 0 \le y \le 1$

$$\therefore T_q = \frac{1}{(n-1)!} P_0 \beta \times$$

$$\left[\sum_{x=n}^{M-1} \frac{1}{n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x \left(\frac{n\beta}{M-1}\right)^{x-n} {}^{M-1}C_{x-n} \int_0^1 \left(\frac{(M-1)y}{n\beta}\right)^{x-n+1} (1-y)^{(M-1)-(x-n)} \frac{(M-1)}{n\beta} dy \right]$$

$$= \frac{1}{(n-1)!} P_0 \beta \left[\sum_{x=n}^{M-1} \frac{1}{n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x \left(\frac{n\beta}{M-1}\right)^{x-n} \left(\frac{M-1}{n\beta}\right)^{x-n+2} {}^{M-1}C_{x-n} \int_0^1 y^{x-n+1} (1-y)^{(M-(x-n)-1)} dy \right]$$

$$= \frac{1}{(n-1)!} P_0 \beta \left[\sum_{x=n}^{M-1} \frac{1}{n^{x-n}}\left(\frac{\alpha}{\beta}\right)^x \left(\frac{n\beta}{M-1}\right)^{x-n} \left(\frac{M-1}{n\beta}\right)^{x-n+2} {}^{M-1}C_{x-n}\, \beta'\big(x-n+2, M-(x-n)\big) \right]$$

$$= \frac{1}{(n-1)!} P_0 \beta \left[\sum_{x=n}^{M-1} \frac{1}{n^{x-n}} \left(\frac{\alpha}{\beta}\right)^{x} \left(\frac{n\beta}{M-1}\right)^{x-n} \left(\frac{M-1}{n\beta}\right)^{x-n+2} {}^{M-1}C_{x-n} \frac{\Gamma(x-n+2)\Gamma(M-(x-n))}{\Gamma(M+2)} \right]$$

$$= \frac{1}{(n-1)!} P_0 \beta \frac{(M-1)^2}{\beta^2 \Gamma(M+2)} \left[\sum_{x=n}^{M-1} \frac{1}{n^{x-n+2}} \left(\frac{\alpha}{\beta}\right)^{x} {}^{M-1}C_{x-n} (x-n+1)!(M-x+n-1)! \right]$$

$$= \frac{1}{(n-1)!} P_0 \frac{(M-1)^2}{\beta(M+1)!} \left[\sum_{x=n}^{M-1} \frac{1}{n^{x-n+2}} \left(\frac{\alpha}{\beta}\right)^{x} {}^{M-1}C_{x-n} \, (x-n+1)!(M-x+n-1)! \right]$$

By using substitution $x = n + r$

$$T_q = \frac{1}{(n-1)!} P_0 \frac{(M-1)^2}{\beta(M+1)!} \left[\sum_{r=0}^{M-n-1} \frac{1}{n^{r+2}} \left(\frac{\alpha}{\beta}\right)^{n+r} {}^{M-1}C_r \, (r+1)!(M-r-1)! \right]$$

$$T_q = \frac{1}{(n-1)!} P_0 \frac{(M-1)^2}{\beta(M+1)!} \left[\sum_{r=0}^{M-n-1} \frac{1}{n^{r+2}} \left(\frac{\alpha}{\beta}\right)^{n+r} \frac{(M-1)!}{r!(M-r-1)!} (r+1)!(M-r-1)! \right]$$

$$T_q = \frac{1}{(n-1)!} P_0 \frac{(M-1)^2}{\beta M(M+1)} \left[\sum_{r=0}^{M-n-1} \frac{1}{n^{r+2}} \left(\frac{\alpha}{\beta}\right)^{n+r} (r+1) \right] \quad (19)$$

By using this relation and Little's formula several others parameters of queuing model can be calculated.

6. Little's Formula:

$$N_s = \alpha' T_s \;\; N_q = \alpha' T_q \quad (20)$$

Where $\alpha' = \alpha_{eff} = \alpha(1 - P_M)$ which is effective average arrival rate

Where N_s is expected number of customers in the system, N_q is expected number of customers in the queue, α' is effective average arrival rate of customer , T_s is the waiting time of customer in the system and T_q is the waiting time of customer in the queue.

7. Performance Measures of Multiple Server Finite Queue Length Queuing Model:

7.1Expected Waiting Time of a Customer in the System:

Expected waiting time of a customer in the system is given by

T_s = Expected waiting time of customer in the queue+ service time of customer

$$= T_q + \frac{1}{\beta} \quad (21)$$

7.2 Expected Number of Customers in the Queue:

Number of customers in the queue is possible only when the arrival customers are greater than the number of server. Therefore we use here probability of case-2.

By using Little's formula (20) we have

Expected number of customers in the queue is given by

$N_q = \alpha' T_q$ Where $\alpha' = \alpha(1 - P_M)$

$$\therefore N_q = \alpha \left(1 - \frac{1}{n! n^{M-n}} \left(\frac{\alpha}{\beta}\right)^{M} P_0 \right) \times T_q \quad (22)$$

7.3 Expected Number of Customer in the System:

The numbers of customers in the system are the customer under service and customers waiting in the queue. Again we use here probability of case-2

By using Little's formula (20) we have

Expected number of customers in the system is given by

$N_s = \alpha' T_s$ Where $\alpha' = \alpha(1 - P_M)$

$$N_s = \alpha\left(1 - \frac{1}{n!n^{M-n}}\left(\frac{\alpha}{\beta}\right)^M P_0\right) \times T_s \tag{23}$$

7.4 Expected Waiting Time of a Customer in the Queue for Busy System:

Expected waiting time of a customer in the queue for busy system is given by

$$T_b = \frac{T_q}{\Pr ob.\ of\ system\ being\ busy} = \frac{T_q}{1 - P_0} \tag{24}$$

7.5 Expected Number of Customers Served Per Busy Period:

Expected number of a customer served per busy period is given by

$$N_b = \frac{N_s}{\Pr ob.\ of\ system\ being\ busy} = \frac{N_s}{1 - P_0} \tag{25}$$

8. Conclusion

An application of Binomial distribution and Uniform distribution gives the time assurance of particular number of customers arrived or departs. Uniform distribution gives the 100% assurance of arrival another customer and 100% served another customer by server in the next particular time limit. The study of different parameters of the model takes place when model works under case-2 condition. Waiting time relation is used in the determination of other remaining parameters of the queuing model from equation (21) to (25). All the effective measures of the model help to study the strength of model.

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