Developing New Model of Constraints Management in Linear Programming problems with reference to optimizing objective function

[1]Patel Snehkumar N. & [2]Dr. Girish Bhimani

[1]*Ph.D. Research Scholar, Department of Statistics, Saurashtra University, Rajkot, Gujarat (India)*
[2]*H.O.D of Department of Statistics, Saurashtra University, Rajkot, Gujarat (India)*

ARTICLE DETAILS

Article History
Published Online: 10 December 2018

Keywords
Intercept matrix, projection value, Constraint selection, linear programming problem (LPP), Cosine criterion, simplex method

Corresponding Author
Email Id.: *shaktipatel619@yahoo.com*

ABSTRACT

Linear programs (LP) play an important role in the theory and practice of optimization problems. Linear programming deals with problems such as maximising profits, minimising costs or ensuring you make the best use of available resources. While formulating a linear programming model, system analyst and researchers often tend to include all the possible constraints, although some of them may not be binding at the optimal solution. It is well known that, for most of the large scale LP problems, only a relatively small percentage of constraints are binding at the optimal solutions. Researchers have proposed methods which identify those constraints most likely to be tight at optimality. This paper proposes a new approach for finding solutions to LP problems by using a part of the constraints with the help of intercept and projection values of the each constraint. This method is more efficient when compared with the existing method. The developed algorithm is implemented by programming language Java and the computational results are presented. It shows that the proposed method reflects a significant decrease in the computational effort and is one of the best alternatives to select the necessary constraints prior to solving an LP problem.

1. Introduction

Linear programming is a tool for solving optimization problems. Many researchers have proposed different algorithms to solve linear programming problems. Solving linear programming problems efficiently has always been a fascinating pursuit for computer scientists and mathematicians. The computational complexity of any linear programming problem depends on the number of constraints and variables of the LP problem. Prepressing is an important technique in the practice of linear programming problem. The reduction of large scale problem can save significant amount of computational effort during the solution of a problem. Many researchers have proposed algorithms for selecting necessary constraints in LP models. In solving an LP Problem (LPP),it is acknowledged that redundancies do exist in most of the practical LPPs. The importance of detecting and removing redundancy in a set of linear constraints is the avoidance of all the calculations associated with those constraints when solving an associated LPP. Many researchers [1 - 15] have proposed different methods for solving and identifying redundant constraints in LPP. LPP represents a mathematical model for solving numerous practical problems and it consists of objective function and a set of constraints.

The general form of LPP is Optimize

$Z = C^T X$ Subject to the constraints,

AX <b X > 0

Where X represents the vector of variables (to be determined) while C and b are vectors of (known) coefficients and A=[aij] is a (known) matrix of coefficients. If the technological coefficients ay's > 0 and b_i> 0, the LPP is said to be "Non-negative linear programming problem (NNLPP)". In solving an LPP, we tend to include all possible constraints that will increase the number of iterations and computational effort. It is well known that for mostof the large scale LP problems, only a relatively small percentage of constraints are binding at the optimal solutions. The purpose of this paper is to propose a new heuristic approach which obtains solutions to the LP problems with the use of some of the constraints.

This paper proceeds as follows: Section 2 gives the definitions of some of the terms and some of the earlier methods for selecting the constraints to solve the LPP. Section 3 presents a proposed approach to identify the constraints from a set of constraints and to solve the problem with some numerical examples. Section 4 gives the computational results. Section 5 concludes the paper.

2. Some Preceding Methods

In this section, the definitions of the terms such as binding and redundant constraints and some of the earlier methods developed for selecting the constraints are explained.

2.1 Definitions

Binding Constraints

These are the constraints which define the feasible region and they are involved in finding the optimal solution. They are also called active/ operative constraints.

Redundant Constraints

A redundant constraint is a constraint that can be removed from a system of linear constraints without changing the feasible region.

Consider the following system of m non-negative linear inequality constraints and n variables (m>n).

AX<b, X>0

where A G R^{mxn}, b G R^m, X G R^n and 0 G R^n.

RRIJM 2015, All Rights Reserved

Let $A_iX < b_i$ be the i^{th}constraint of the system (1) and let S = {XGR^n/$A_iX<b_i$,X>0}

be the feasible region associated with system (1).

Let S_k = {XGR^n/ $A_iX<b_i$, X>0,i^k} be the feasible region associated with the system of equations AiX<bi,

i = 1, 2, m, i^k. The k^{th} constraint $A_kX<b_k$(1<k<m) is redundant for the system (1) if and only if S =Sk

2.2 Largest Summation in rule

In this approach, each (constraint is divided by the corresponding right-hand side value and denoted by *~y~j* Compute Si = **£™$_{=1}$|aVu|** optimize with the constraints corresponding' V.T* the first two maximum values of Sj. If it is not optimal, append the violated constraints with the existing problem.

2.3 Cosine Simplex Algorithm

For a given problem it begins by solving a relaxed problem consisting of the original objective function subject to a constraint corresponding to the index. Empty bounded feasible region. At each subsequent iteration of the algorithm, the most parallel constraint to the objective function among those constraints violated by the solution to the current relaxed problem is appended to it. When no constraints are violated, the solution of the current relaxed problem is optimal to the original problem.

2.4 Constraints Optimal Selection Technique

They have taken NNLPP and these two factors are taken into consideration (i) Angle of its normal column vector aj^T with c of the objective function, (ii) The depth of the cut that a $_j$x < bj removes as a violated inoperative constraint from the feasible region.

Define RAD ($_{ai}$, b_{if} c) =^cos($_{ai}$,c)

Take the constraint corresponding to max RAD (a_{ji} bj c) and set the initial relaxed problem. On solving we get a solution. Check for inoperative constraints in the decreasing order. Take the first one violated by the solution and append the constraint to relaxed problem. Continue this till no inoperative constraint exists. The resultant solution is the optimal solution.

2.5 Limitations

The cosine criterion [3] plays a vital role more in the small scale problem than in the simplex but not in the large scale problem. The number of constraints selected is more in the large scale problem. The COSTs is more efficient than cosine simplex but when tie appears it undergoes cycling. Moreover, COSTs is for NNLPP whereas this new proposed method withstands all these limitations.

3. Proposed Method

This Proposed method identifies the relevant set of constraints which finds out the optimal solution for the following LPP.

Maximize Z = C^TX

Subject to the constraints

AX < b

X > 0, A e R^{mxn}, b > 0, X, C e R^n

3.1 Algorithm

The steps of the algorithm are as follow:

1. Construct a matrix of intercepts of all the decision variables formed by each of the resource constraints along the respective coordinate axis

Gji = —^{L}, ajj > 0, i = 1,2...m andj = 1,2,...,n.

Find the min {Gji } = [3j in each row j.

2. Set L = {i : *mm$_{aUeastoneJ}${e$_n$}* U < *[$$_n$* < max /?;}, *for all j),* L is the index of the selected set of constraints and assumes it contains p number of constraints, p<m.

3. Set J = { m + j , j = 1,2,3,...,n}
4. Find a_K= T^:, K e L U J .
5. Set q = 1.

Solve the relaxed problem Maximize Z = C^TX Subject to the constraints $a_rx < b_r a_s x < b_s$ x > 0 ,

where the r^{th} and s^{th}constraints are selected by *r — org* ($max_{fc}a_k$) and s — arg($min_{fe}a_{fc}$^, provided ak>0. Let the solution obtained, be x (q). Check for violated constraints. If none is found, stop. Since x(q) is the optimal solution. Otherwise

6. Take l = arg(max a_k)of the violated constraints if q is odd or Take

l = arg(min a_k)of the violated constraints if q is even. Set q=q+1.Append the violated constraint corresponding to index l to the relaxed problem and go to step 5.

3.2 Illustration

The Proposed method is illustrated with the following numerical examples.

Example 1:

Max Z = $20x_1 + 10x_2+x_3$ Subject to the constraints

3x1-3x2+5x3< 50

$1x_1+0x_2+1x_3 < 10$

1x1-1x2+4x3 < 20

2x1+1x2+3x3 < 30

2x1+2x2+1x3<66 1x1+4x2+3x3<75 $2x_1+2x_2+3x_3 < 80$

Solution:

Decision variables	Si	S2	S3	S4	S5	S6	S7	Pj
x1	16.67	10	20	15	33	75	40	10
x2	-	-	-	30	33	18.75	40	18.75
x3	10	10	5	10	66	25	26.67	5

Here the set L={1, 2, 3, 4, 6} are the selected constraints.
*ai= 5.337 <*2= 14.849 a3=3.3 a4=14.165 a6=12.355 a8=-20* a_9*=-10* a_1o*=-1* ***Here we select*** a_i*> 0 only.*
q=1

Optimise the objective function subject to 2^n and 3^r constraints; here the solution
is not optimal.
q=2
Select 2nd, 3rd and 4th constraints, on solving we obtain the optimal solution to be x_1=10, x_2=10, x_3=0;Z=300 ;

Example 2:
Max Z = 5x1+6x2+3x3 Subject to the constraints
5x1+5x2+3x3 <50
$2x_1+2x_2+0x_3 < 40$
$7x_1+ 6x_2-9x_3 < 30$
$5x_1 + 5x_2 + 5x_3 < 35$
12x1 + 6x2 - 1x3 < 90
$-1x_1+ 1x_2- 2x_3 < 20$
x1,x2,x3 > 0

Solution:

Decision variables	S1	S2	S3	S4	S5	S6	Pi
x1	10	20	4.29	7	7.5	-	4.29
x2	10	20	5	7	15	20	5
x3	16.67	-	-	7	-	-	7

Here the set L={3,4} are the selected constraints.

a3=3.415 a4= 8.083 013= -0.4082 014=-3 as=-2 ae=-5 Here we select **aj> 0 only.**
q=i
Optimise the objective function subject to 3^r and 4^t constraints. We obtain the
optimal solution to be
x1=0, x2=6.2, x3=0.8;Z=3.96 ;

Example 3:
Max Z = x1+x2+x3
Subject to the constraints $12x_1+12x_2+13x_3<312$
-11x1+14x2+15x3<1155 $8x_1+7x_2+20x_3<200$
-4x1+3x2+6x3<900 $2x_1+3x_2+7x_3<70$ 9x1+15x2+2x3<120
$-1x_1+3x_2+8x_3<70$
5x1-2x2+7x3<75 $4x_1+6x_2 3x_3< 50$

Solution:

Decision variables	S1	S2	S3	S4	S5	S6	S7	S8	S9	PJ
x1	26	-	25		35	13.3		15	12.5	12.5
x2	26	82.5	28.6	300	23.3	8	23.3		8.3	8
x3	24	77	10	150	10	60	8.75	10.7		8.75

Here the set L= {3,5,6,7,8,9} are the selected constraints.
a3= 1.545 a5= 1.524 ae=1.477 a7=1.162 a8=1.132 a9=0.896
a10= aii= ai2 = -1 Here we select **ai> 0 only.**
q=1
Optimise the objective function subject to 3^r and 9^t constraints, solution is x1=15.38 , x2=0 , x3=3.85;Z=19.23 ; But it is not optimal. Here 6th& 8' Constraints are In-operative.
q =2
Select 3^r , 9^t and 6^t constraints, on solving solution is xi=12.2 , x_2=0 , x_3=5.12;Z=17.32 ; But it is not optimal. Here 8th Constraint alone is In-operative. **q = 3**
Select 3rd, 9th,6th,8th constraints, on solving we obtain the optimal solution to be x1=7.6, x2=2.63, x3=6.04;Z=16.27 ;

4. Computational Results

The efficiency of the algorithm is tested by solving LPP for small scale problems using the Cosine Simplex method and the proposed method. The following table 1shows a comparison between the numbers of constraints selected for two methods and it shows that the proposed method is very useful to solve the LPP with a minimum amount of computational efforts and time. The comparative results of the two methods are clearly shown in the respective graph followed by the table 1. Table 2 and 3 shows the number of constraints selected and time in seconds for the proposed method only. The Cosine simplex method is applicable for small-scale problems only.

Table 1. Comparison of the Computational Efforts in Solving LPP: Small- scale Problems

S.No.	Size of the problem		No. of Selected Constraints.			No. of Iterations	
	No. of. Constraints (m)	No. of. Variables (n)	Simplex Method	Cosine Simplex Method	Proposed Method	Cosine Simplex Method	Proposed Method
1	3	3	3	2	2	5	3
2	4	2	4	4	1	11	2
3	5	4	7	2	2	4	2
4	4	6	4	4	3	20	9

RRIJM 2015, All Rights Reserved

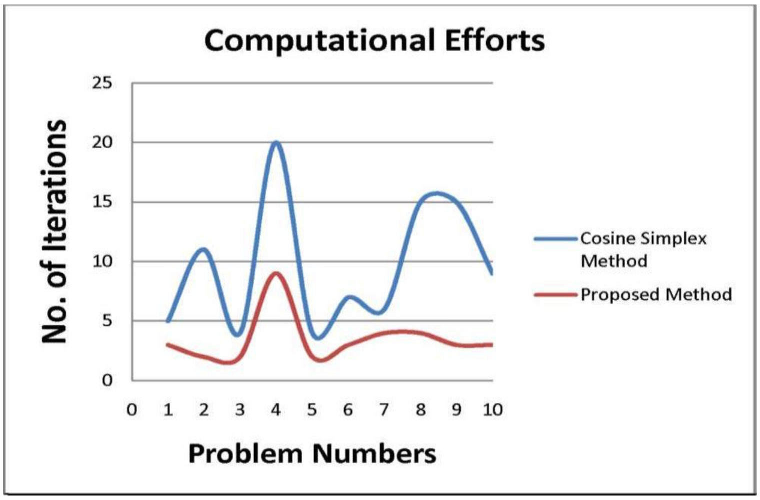
5	4	3	4	2	2	4	2
6	6	3	6	3	2	7	3
7	3	7	3	2	2	6	4
8	7	10	7	4	2	15	4
9	4	5	4	4	2	15	3
10	5	2	5	3	2	9	3

Table 2. Comparison of the Computational Efforts in Solving LPP: Large - Scale Problems

S.NO.	File name	No. of Constraints (m)	No. of Variables (n)	Proposed Method	
				No. of Selected Constrains	Time (Seconds)
1	Scpe1	50	500	39	55
2	Scep2	50	500	26	26
3	Scep3	50	500	26	23
4	Scep4	50	500	28	26
5	Scep5	50	500	40	51

Table 3. Comparison of the Computational Efforts in Solving LPP: Netlib Problems

S.NO.	File name	No. of Constraints (m)	No. of Variables (n)	Proposed Method	
				No. of Selected Constrains	Time (Seconds)
1	afiro	20	20	18	4
2	fit1d	24	24	23	4
3	fit2d	25	25	18	3
4	sc50b	28	28	24	5
5	sc50a	29	29	25	4
6	kb2	39	39	38	7
7	vtpbase	51	51	40	9
8	bore3d	52	52	51	14



5. Conclusion

A new algorithm using an intercept and pivot index values has been proposed for the selection of constraints to solve the linear programming problems. The resulting model is solved in order to establish the validity of the proposed method. A significant reduction in the computational effort is achieved.

References

1. A.Boneh, S.Boneh and R.J.Caron, Constraint classification in mathematica l programming, Mathematical Programming, 61(1993), 61-73.
2. J.C.G.Boot, On trivial and binding constraints in programming problems, Management Science, 8(1962), 419-441.
3. H.W.Corley, Jay Rosenberger, Wei-Chang Yeh and T.K.sung, The Cosine Simplex Algorithm, The International Journal of advanced manufacturing Technology, 27(2006), 1047-1050.
4. Danny C Myers, A dual simplex Implementation of a constraint selection algorithm for linear Programming, The

Journal of the operational research Society, 43(1992), 177-180.

5. Danny C Myers and Wei Shih, A Constraint selection technique for a class of linear programs, Operation Research letters, 7(1988), 191-195.
6. N.Karmarker, A new polynomial-time algorithm for linear programming, Combinatorica, 4(1984), 373-395.
7. K.G.Murty and Y.Fathi, A feasible direction method for linear programming, Operations Research Letters, 3(1984), 121-127.
8. S.Paulraj, C.Chellappan and T.R.Natesan, A heuristic approach for identification of redundant constraints in linear programming models, International Journal of computer mathematics, 83(2006), 675-683.
9. S.Paulraj andP.Sumathi, A Comparative Study of Redundant Constraints Identification Methods in Linear Programming Problems,Mathematical Problems in Engineering, 2010(2010), Article ID 723402, 16pages.
10. N.V.Stojkovic and P.S.Stanimirovic, Two direct methods in linear programming, European Journal of operational research,131(2001), 417-439.
11. P.Sumathi and S.Paulraj Identification of Redundant Constraints in Large Scale Linear Programming Problems with Minimal Computational Effort, Applied Mathematical Sciences, 7(2013), 3963-3974.
12. H.A Taha, Operations Research, An Introduction, 8th Edition, Prentic hall of India, New Delhi, (2006).
13. Wei - Chang Yeh and H.W.Corley, A simple direct cosine simplex algorithm, Applied mathematics and computation, 214(2009), 178-186.
14. Wei Li, Haohao Li, On Simplex method with most-obtuse-angle rule and Cosine rule, Applied Mathematics and Computation, 217(2011),7867-7873.
15. Y.Yamamoto, A new method for solving a Linear programming problem, 49th IEEE conference on decision and control, December 15-17, (2010) 3891-3895.