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The Compactness of fuzzy metric spaces using various topology

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ABSTRACT
We studied that the topology induced by any fuzzy metric space is metrizable we also show that every separable fuzzy metric space admits a precompact fuzzy metric and that a fuzzy metric space Ps compact if and only if it is precompact and complete.

1. Introduction

The theory of fuzzy sets was introduced by L. Zadeh in 1965. Many authors have introduced the concept of fuzzy metric spaces in different ways. In this paper; we modify the concept of fuzzy metric space introduced by Kramosil and Michalek and define a Hausdroff topology on this fuzzy metric space.

We show that every metric induces a fuzzy metric. Further we modify the definition of Cauchy sequence in, because of the fact that even R is not complete with the definition given in.

We define F-bounded ness on a fuzzy metric space and prove that compactness implies F-bounded ness. We prove that every closed ball is a closed set in a fuzzy metric space.

Finally: we prove Baire's theorem for fuzzy metric spaces.

2. Topology Induced by a Fuzzy Metric

Topology induced by a Fuzzy metric is defined by the following way-

Definition-Let (x : m : *) be a fuzzy metric space we define open ball B (x : r : t) with centre

$$x \in X$$

and radius r; $o < r < 1 : t > o$ as

$$B (x : r : t) = \{ y \in x : m (x : y : t) > 1- r \}$$

Definition-Let (X : M : *) be a fuzzy metric space. A subset A of X is said to be F-

bounded if and only if there exist

$t > o$ and $o < r < l$

such that

$m (x : y : t) > 1\text{-}r$ for all $x. y \in A$.

Remark-Let (x : m : *) be a fuzzy metric space induced by a metric d on x.

then $A \subseteq x$ is F-bounded if and only if it is bounded.

Definition-A sequence $\{x_n\}$ in a fuzzy metric space (x : m : *) is a Cauchy sequence if and only if lim m $(x_{n+p} : x_n : t) = 1 : \quad P > o : t : > o\ n \to \infty$

A fuzzy metric space in which every Cauchy sequence is convergent is called a complete fuzzy metric space.

Note-We note that with the above definition: even R fails to be complete. For example.

Consider

$$S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots\ldots\ldots\ldots + \frac{1}{n}$$

in (R:M): where

$$m (x:y: t) = \frac{t}{t + d(x:y)}$$

a is a metric on R.

Now

$M (S_{n+p} : S_n : t)$

$$= \frac{t}{t + |S_{n+p} - S_n|}$$

Now.

$$= \frac{t}{t + (\frac{1}{n} + 1) + (\frac{1}{n} + 2) + \ldots\ldots(\frac{1}{n} + P)}.$$

Therefore

$$\lim_{n \to \infty} m (S_{n+p} : S_n : t) = 1$$

thus $\{S_n\}$ is a Cauchy sequence in the fuzzy metric space R. if R is fuzzy complete then there exits

$x \in R$

such that

$m (S_n : x : t) \to 1$

as $n \to \infty$

From this if follows that

$\frac{t}{t + | S_n - x |} \to 1$ as $n \to \infty$

Further,

$| S_n - x | \to 0$ as $n \to \infty$

And so $\to x$ in R which is not true.

Hence to make R complete fuzzy metric space we redefine Cauchy sequence as follows.

Definition- A sequence $\{x_n\}$ in a fuzzy metric space $(x : M *)$ is a Cauchy sequence if and only if for each $\varepsilon > o$; $t > o$ there exits

$n_o \in N$

such that

$m (x_n : x_m : t) > 1-\varepsilon$ for all $n : m \geq n_o$.

Definition- Let $(x : m : *)$ be a fuzzy metric space. Then we define a closed ball

with centre

$x \in X$

and radius $r: o < r < 1$: $t > o$ as

$B [x : r : t] = \{ y \in X ; M (x : y : t) \geq 1- r \}$.

3. Fuzzy Compactness

In this Section we define fuzzy compactness.

Definition- A fuzzy metric space $(x : m : *)$ is called precompact if for

each r; with $o < r < 1$ and each $t > o$

There is a finite subset of A of X: such that $X = \cup_{a \in A} B (a : r : t)$.

in this case we say that M is a precompact fuzzy metric on X.

A fuzzy metric space $(x : m : *)$ is called compact if $(x: T_m)$ is a compact topological space.

Lemma - A fuzzy metric space is precompact if and only if every sequence has a Cauchy sequence.

Proof- Suppose that $(x : m : *)$ is a precompact fuzzy metric space. Let (x_n) $n \in N$ be a sequence in X. for each $m \in N$ there is a finite subset A_m of X such that

$$X = \cup_{a \in A_m} B(a : \frac{1}{m} : \frac{1}{n}).$$

hence : for m=1; there exists an $a_1 \in A_1$ and a subsequence $(x_{1(n)})_{n\in N}$ of (x_n) $n\in N$ such that

$x_1(n) \in B(a_1:1:1)$ for every $n\in N$.

similarly there exits a_n $a_2 \in A_2$ and a subsequence $(x_2(n))$ $n\in N$ of $(x_{1(n)})$ $n \in N$ such that

$$X_2(n) \in B\left(a_2:\frac{1}{2}:\frac{1}{2}\right)$$

For every $n\in N$.

Following this process:

For $m \in N$; $m > 1$ there is a_n $a_m \in Am$ and a Subsequence $(x_{m(n)})$ $n\in N$ of

$(x_{(m-1)(n)})$ $n\in N$ such that

$X\,m(n) \in B.\left(a_m:\frac{1}{m}:\frac{1}{n}\right)$ for every $n\in N$.

now consider the subsequence $(x_{n(n)})$ $n\in N$ of (x_n) $n\in N$. given r with o<r<1 and t > o there is an $n_o \in N$ such that

$$(1-(1/n_o)) * (1-(1/n_o))>1-r$$

And $\frac{2}{n_o} < t$.

Then for every K: $m \geq n_o$ we have

$m(xk(k): xm(m);\ t)$

$\geq m(x_{k(k)}; x_{m(m)}; 2/n_o$
$\geq m(x_{k(k)} : a_{no}: 1/n_o)$

$* \; m(a_{no}: x_{m(m)}: 1/n_o)$

$\geq (1-1/n_o * (1-1/n_o) > 1-r$

Hence $(x_{n(n)})$ $n\in N$ is a Cauchy sequence in (x : m : ∗) Conversely : suppose that (x : m : ∗) is a non- precompact fuzzy metric space. Then there exist r ;

With o<r<1 and t >o.

Such that for each finite subset A of X; $X = \cup a \in A\; B(a : r: t)$ Fix $X_1 \in X$ there is

$X_2 \in X\setminus B(x_1 : r : t)$.

Moreover : there is

$X_3 \in X \setminus \cup^2 k=1\ (x_k:r:t)$.

Following this process we construct a sequence (x_n) $n\in N$ of distinct points in x: such that

$Xn+1 \notin \cup^n_{k=1} B(x_k : r : t)$ For every

$n\in N$.

Therefore (x_n) $n\in N$ has no Cauchy subsequence.

Theorem- A Fuzzy metric space is compact if and only if it is precompact and complete.

Proof suppose that (x:m:∗) is a complete fuzzy metric space for each r ; with o< r <1 and each t >o the open cover

{ B(x:r:t): $x\in X$} of X: has a finite sub cover. Hence (x:m:∗) is precompact.

On the other hand: every Cauchy sequence (x_n) $n\in N$ in (x:m:∗) has a cluster point $y\in x$.

And all (x : m : ∗) be a fuzzy metric space. If a Cauchy sequence clusters to a point $x \in x$: there the sequence converges to x.

Then (x_n) $n\in N$ converges to y. thus (x:m:∗) is complete.

Conversely let (x_n) $n\in N$ be a sequence in X and from the lemma as discuses before and the completeness of (x:m:∗) if follows that (x_n) $n\in N$ has a cluster point.

Since by $(x:T_m)$ is metrizable and every sequentially compact metrizable space is compact we conclude that (x:m:∗) is compact.

4. Conclusion

In this work we have discussed fuzzy metric space. We shown that the topology induced by any fuzzy metric space is metrizable. Firstly we define a fuzzy metric space with the help of a lot definitions and theorems. we have also discussed Hausdroff topology on a fuzzy metric space and at last compactness of a fuzzy metric space.

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