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Optimal Solution of Two Stage Solid Transportation Problem with Varying Cost

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ABSTRACT

In this paper, we develop a solution procedure for solving two stage solid transportation problem. In certain circumstances, due to some restrictions (e.g. storage constraint), destinations are not able to get the quantity in excess of their maximum storage. Thus a single shipment is not possible to supply the full demand. Therefore items are shipped in more than one stage. In the present paper, we have discussed two stage solid transportation problem in which the transportation cost in second stage is proportionally increased by the suppliers due to fuel, toll or octroi charges etc. Numerical example is illustrated in support of the theory.

Mathematical Subject Classifications (2010): 90B06, 90C05

1. Introduction

Transportation problem was first formulated in 1941, by Hitchcock [4], is a special case of linear pro-gramming problem in which our objective is to satisfy the demand at destinations from the supply at the minimum transportation cost.It was further developed in 1949, by Koopman [5] and in 1951, by Dantzig [2].

The traditional transportation problem is well-known optimization problem in operational research, in which two kinds of constraints are taken into consideration, i.e. source constraints and destination con-straints. But in the real system, conveyance plays a very important role in determining the cost of transportation. So conveyance should also be taken into account while finding the optimum cost of transportation. For such case the transportation problem turns into solid transportation problem. Shell [10] was the first person who introduced the concept of solid transportation problem in 1955 and it was first solved by Haley [3] in 1962, by using extension of MODI method.

In 1997, Li and Kobuchi [6] have proposed neural network approach for multicriteria solid transporta-tionproblem.Authors have suggested a neural network architecture to solve single-objective solid trans-portation problem according to augmented Lagrange multiplier method. In 2007, Basu et al [1] have developed a new algorithm for finding solution of solid fixed charge transportation problem. In 2010, Pandian and Anuradha [7] gives min zero - min cost method for solving solid transportation problem without using MODI method.Similarly in 2016, plain line method for solving solid transportation prob-lem was given by Pandian and Kavitha [8] in which they find optimal solution directly without using MODI method.

In 2016, Pandian and Kavitha [9] solve two stage solid transportation problem in which the cost of trans-portation is same in both stages.They have proved that in such cases after satisfying the requirement at the first stage, the total cost of transportation of first stage and any one of the second stage would be same as the whole quantity would be transported in one shift.But in real life due to certain reasons(e.g. labour, fuel, toll and VAT charges etc.) the cost of transportation may not remain same in the second stage.Therefore in second shift supplier increases the cost of transportation. In the present paper we have considered the situation when the supplier proportionally increase the cost at the second stage. We use maxmin principle to show that in such cases the total transportation cost (both stages) will be minimum in that case where the cost of transportation is maximum in the first stage.

This paper is organized as follows: Section 2, contains Mathematical Formulation. In Section 3, Pro-posed method is given. Numerical examples are given in Section 4, to illustrate the algorithm. In section 5,Conclusion is given.

2. Mathematical Formulation

2.1 Solid Transportation Problem

Solid transportation problem is formulated as:

$$(O)\ \text{Min } z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} c_{ijk} t_{ijk}$$

subject to

$$\sum_{j=1} \sum_{k=1}^{p} t_{ijk} = a_i, i=1,2,\ldots,m,$$

$$\sum_{i=1} \sum_{k=1}^{p} t_{ijk} = b_j, j=1,2,\ldots,n,$$

$$\sum_{i=1}^{n} \sum_{j=1} t_{ijk} = e_k, k=1,2,\ldots,p,$$

$$x_{ijk} \geq 0, \text{ for all i, j and k.}$$

Where a_i is the amount of the material available at i^{th} source, b_j is the amount of the material re-quired at j^{th} destination, e_k is the amount of the material transported by k^{th} conveyance, c_{ijk} is the unit cost of transportation from the i^{th} source to j^{th} destination by means of the k^{th} conveyance and x_{ijk} is the number of units to be transported from i^{th} source to j^{th} destination by means of the k^{th} conveyance.

m n p

If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{p} e_k$, then we say it is balanced solid transportation problem. Otherwise, it is called unbalanced.

Because of the special structure of the transportation model, the problem can also be represented as Table 1.

Table 1: Results (IBFS by using existing methods and Optimal Solution)

										Capacity
Conveyance	C_1			C_1			C_1			e_1
		..			..			..		..
			C_p			C_p			C_p	e_p
	D_1			::			D_n			Supply
S_1	c_{111}	..	..	..	..	..	..	..	..	a_1
..	..	..	..	..	..	..	..	..	..	..
S_m	c_{m11}	..	..	..	..	..	..	..	c_{mnp}	a_m
Demand	b_1			..			b_n			

Two stage STP can be formulated as:

Min $z = z_1 + z_2$, where z_1 and z_2 are respectively cost of tranportation in first and second stage,

subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} (c_{ijk}\, x_{ijk}) \le z_1 \qquad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} (r c_{ijk} y_{ijk}) \le z_2$$

$$\sum_{j=1}^{n}\sum_{k=1}^{p} (x_{ijk}) \le a_i \;, i = 1, 2, \dots, m \qquad \sum_{i=1}^{m}\sum_{k=1}^{p} (x_{ijk}) = k_j, \; j = 1, 2, \dots, n$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} (x_{ij}) = t_k, \; k = 1, 2, \dots, p \qquad \sum_{j=1}^{n}\sum_{k=1}^{p} (y_{ijk}) = a_i - \sum_{j=1}^{n}\sum_{k=1}^{p} (x_{ijk}) \;, i = 1, 2, \dots, m$$

$$\sum_{i=1}^{m}\sum_{k=1}^{p} (y_{ijk}) = b_j - k_j \;, j = 1, 2, \dots, n \qquad \sum_{i=1}^{m}\sum_{j=1}^{n} (y_{ijk}) = v_k - t_k, \; k = 1, 2, \dots, p, \qquad x_{ijk}, y_{ijk} \ge 0, \text{ and } r > 1.$$

Where k_j is the minimum requirement (maximum storage capacity) of the j^{th} destination, t_k is the min-imum (maximum amount of the) quantity shipped by the k^{th} conveyance. x_{ijk} and y_{ijk} are respectively the amount of quantity shipped from the i^{th} origin to j^{th} destination by k^{th} conveyance in stage 1 and 2.

3. Proposed method

Theorem 1.Maxmin Principle: It states that an optimal solution of two stage STP can be obtained from optimal solution of single stage STP in such a way that among all the alternatives of z_1 satisfying the given constraints obtained from optimal solution of single stage STP the one which gives maximum value of z_1 will minimize the total transportation cost of two stage STP.

Proof:First of all we will find optimal solution of original problem (O).

Let the optimal value of z is Z, and $Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} (c_{ijk} T_{ijk})$

Among all the alternatives of z_1 of the problem (F) obtained from the solution Z of the problem (O) and satisfying the given constraints,

Let the one which maximize z_1 is Z_1.

Now $z_2 = z - z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} (r c_{ijk} y_{ijk})$

$= \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} (r c_{ijk}(t_{ijk} - x_{ijk})) = r(z - z_1)$

From above equation it is clear that z_2 will be minimum when z is minimum and z_1 is maximum.

3.1 Algorithm of Proposed Method

To proceed with proposed method the given steps are followed:

step 1. Represent the given STP into the form of cost matix as Table 1 step 2. Balance the given STP, if it is not balanced by adding dummy supply/demand/conveyance as per requirement.

step 3. Find optimal solution of the given STP.

step 4. Split the original problem in two stages and name the objective functions as z_1 and z_2.

step 5. Obtain all alternatives of z_1 from value of z obtained from step 3.

step 6. Obtain the maximum value ofz_1 from step 5 .

step 7. Calculate value of z_2 from values of z and z_1 obtained from step 3 and 6 respectively.

4. Numerical Examples

Numerical example taken from [9]:In a paramedical company a product is produced in three factories and it is send to three destinations by three different modes of conveyances.The unit shipping cost, sup-ply , demand and conveyance are given in table 2.

Table 2: Input data

										Capacity
Convenience	C1			C1			C1			11
		C2			C2			C2		14
			C3			C3			C3	9
	D_1			D_2			D_3			supply
S1	4	7	8	3	9	7	6	7	2	11
S2	4	2	6	1	3	8	8	4	5	13
S3	8	1	3	4	7	3	5	6	4	10
Demand	7			15			12			

with minimum requirement (maximum capacity) of the destinations are $k_1 = 5$, $k_2 = 10$ and $k_3 = 12$.In the second stage the each cost is doubled. Find the minimum cost of transportation in two stages.

First we find the optimal solution to the given STP without considering the minimum demand constraints, using any of the existing methods. Solution is given in table 3.

Table 3: Optimal Solution

Ex.	Obtained Allocations	Optimal Cost
1	$x_{121} = 2$, $x_{133} = 9$, $x_{221} = 9$, $x_{222} = 4$, $x_{312} = 7$, $x_{332} = 3$	70

Now, since maximum capacity of destinations are 5, 10 and 12 respectively. So there are 14 possibilities of transporting the goods in two stages, which are shown in table 4:

Table 4: optimal solution

Case No.	Obtained Allocations and Total Cost in Stage 1	Obtained Allocations and Total Cost in Stage 2	Total Cost in both stages
1	$x_{312} = 5$, $x_{121} = 1$, $x_{221} = 9$, $x_{222} = 0$, $x_{133} = 9$, $x_{332} = 3$ and total cost 53	$x_{312} = 2$, $x_{121} = 1$, $x_{221} = 0$, $x_{222} = 4$, $x_{133} = 0$, $x_{332} = 0$ and total cost 34	87.
2	$x_{312} = 5$, $x_{121} = 2$, $x_{221} = 8$, $x_{222} = 0$, $x_{133} = 9$, $x_{332} = 3$ and total cost 55	$x_{312} = 2$, $x_{121} = 0$, $x_{221} = 1$, $x_{222} = 4$, $x_{133} = 0$, $x_{332} = 0$ and total cost 30	85.
3	$x_{312} = 5$, $x_{121} = 0$, $x_{221} = 9$, $x_{222} = 1$, $x_{133} = 9$, $x_{332} = 3$ and total cost 53	$x_{312} = 2$, $x_{121} = 2$, $x_{221} = 0$, $x_{222} = 3$, $x_{133} = 0$, $x_{332} = 0$ and total cost 34	87.
4	$x_{312} = 5$, $x_{121} = 1$, $x_{221} = 8$, $x_{222} = 1$, $x_{133} = 9$, $x_{332} = 3$ and total cost 55	$x_{312} = 2$, $x_{121} = 1$, $x_{221} = 1$, $x_{222} = 3$, $x_{133} = 0$, $x_{332} = 0$ and total cost 30	85.
5	$x_{312} = 5$, $x_{121} = 2$, $x_{221} = 7$, $x_{222} = 1$, $x_{133} = 9$, $x_{332} = 3$ and total cost 57	$x_{312} = 2$, $x_{121} = 0$, $x_{221} = 2$, $x_{222} = 3$, $x_{133} = 0$, $x_{332} = 0$ and total cost 26	83.
6	$x_{312} = 5$, $x_{121} = 0$, $x_{221} = 8$, $x_{222} = 2$, $x_{133} = 9$, $x_{332} = 3$ and total cost 55	$x_{312} = 2$, $x_{121} = 2$, $x_{221} = 1$, $x_{222} = 2$, $x_{133} = 0$, $x_{332} = 0$ and total cost 30	85.
7	$x_{312} = 5$, $x_{121} = 1$, $x_{221} = 7$,	$x_{312} = 2$, $x_{121} = 1$, $x_{221} = 2$,	83.

	x_{222} = 2, x_{133} = 9, x_{332} =3 and total cost 57	x_{222} = 2, x_{133} = 0, x_{332} =0 and total cost 26	
8	x_{312} = 5, x_{121} = 2, x_{221} = 6, x_{222} = 2, x_{133} = 9, x_{332} =3 and total cost 59	x_{312} = 2, x_{121} = 0, x_{221} = 3, x_{222} = 2, x_{133} = 0, x_{332} =0 and total cost 22	81.
9	x_{312} = 5, x_{121} = 0, x_{221} = 7, x_{222} = 3, x_{133} = 9, x_{332} =3 and total cost 57	x_{312} = 2, x_{121} = 2, x_{221} = 2, x_{222} = 1, x_{133} = 0, x_{332} =0 and total cost 26	83.
10	x_{312} = 5, x_{121} = 1, x_{221} = 6, x_{222} = 3, x_{133} = 9, x_{332} =3 and total cost 59	x_{312} = 2, x_{121} = 1, x_{221} = 3, x_{222} = 1, x_{133} = 0, x_{332} =0 and total cost 22	81.
11	x_{312} = 5, x_{121} = 2, x_{221} = 5, x_{222} = 3, x_{133} = 9, x_{332} =3 and total cost 61	x_{312} = 2, x_{121} = 0, x_{221} = 4, x_{222} = 1, x_{133} = 0, x_{332} =0 and total cost 18	79.
12	x_{312} = 5, x_{121} = 0, x_{221} = 6, x_{222} = 4, x_{133} = 9, x_{332} =3 and total cost 59	x_{312} = 2, x_{121} = 2, x_{221} = 3, x_{222} = 0, x_{133} = 0, x_{332} =0 and total cost 22	81.
13	x_{312} = 5, x_{121} = 1, x_{221} = 5, x_{222} = 4, x_{133} = 9, x_{332} =3 and total cost 61	x_{312} = 2, x_{121} = 1, x_{221} = 4, x_{222} = 0, x_{133} = 0, x_{332} =0 and total cost 18	79.
14	x_{312} = 5, x_{121} = 2, x_{221} = 4, x_{222} = 4, x_{133} = 9, x_{332} =3 and total cost 63	x_{312} = 2, x_{121} = 0, x_{221} = 5, x_{222} = 0, x_{133} = 0, x_{332} =0 and total cost 14	77.

It is clear from maxmin principle that optimal cost is Rs.77 obtained in last case in which z_1 is maximum.

5. Conclusion

In practical life where the acceptor cannot store quantity in excess of their maximum capacity, they will have to pay more to the supplier as they will purchase the goods in more than one shift. The present paper provides a better way in which the acceptor should demand the product so as to minimize the total transportation cost in two stages. The method is easy to understand and simple to apply for the stakeholders.

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