Volume-03
Issue-09
September-2018

ISSN: 2455-3085 (Online)
RESEARCH REVIEW International Journal of Multidisciplinary
www.rrjournals.com [UGC Listed Journal]

Asymptotic Approximations of the Mexican Hat Wavelet Transform for Large Value of Dilation Parameter

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ARTICLE DETAILS

Article History
Published Online: 07 September 2018

Keywords
Asymptotic Approximation, Wavelet Transform, Mellin Convolution, Integral Transform, Dilation, Mexican Hat wavelet transform

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ABSTRACT

In this paper we derive the asymptotic approximations of the Mexican Hat wavelet transform by using the previous results extension of asymptotic approximations of continuous wavelet transform.

1. Introduction

In many applications to physical and engineering problems, it is more important at least from a computational viewpoint to work with the asymptotic approximations of the Mexican Hat wavelet transform. From a physical point of view and these coherent states are of great interest and have several important applications in different disciplines such as mathematics, physics and engineering fields. Although time-frequency analysis of signals had its origin almost so many years ago, there has been major development of the time-frequency distributions approach in the last two decades. The basic idea of the method is to develop a joint function of time and frequency known as a time- frequency distribution that can be described the energy density of a signal simultaneously in both time and frequency domain. In principle, the time-frequency distributions characterize phenomenon in a two dimensional time-frequency plane. Basically, there are two kinds of time- frequency representations. One is the quadratic method covering the time- frequency distributions and the other is the linear approach including the Gabor transform, the Zak transform and the wavelet transform analysis [11]. As Tingbo Hou*, Hong Qin [12] has systematically studies the well-known Mexican Hat Wavelet on Manifold geometry, including its derivation, properties, transforms and applications and J.Gonzalez-Nuevo, F. Argueso, M.Lopez-Caniego, L.Toffolatti, J.L.Sanz, P.Vielva, D.Herranz [13] has proposed a new detection technique in the plane based on an isotropic wavelet family. This family is naturally constructed as an extension of the Gaussian-Mexican Hat wavelet pair and for that reason we call it the Mexican Hat wavelet Family (MHWF).

The continuous wavelet transform of a function $f \in L^2(R)$ with respect to the wavelet $\emptyset \in L^2(R)$ is defined by [1]

$$(W_\emptyset f)(t,s) = \frac{1}{\sqrt{S}} \int_{-\infty}^{\infty} f(u)\, \overline{\emptyset\left(\frac{u-t}{s}\right)}\, du, \quad s > 0, t \in R, \qquad \textbf{(1)}$$

provided the integral exists. The asymptotic expansion for Mellin convolution [1]

$$I(\upsilon) = \int_0^{\infty} f(u) g(\upsilon u) du, \text{ as } \upsilon \to 0^+, \qquad \textbf{(2)}$$

Let us rewrite **(1)** in the form [1]:

$$(W_\emptyset f)(t,s) = \vartheta^{\frac{1}{2}} \int_{-\infty}^{\infty} f(u+t)\, \overline{\emptyset(\vartheta u)}\, du$$

$$= \vartheta^{\frac{1}{2}} \left\{ \int_0^{\infty} f(u+t)\, \overline{\emptyset(\vartheta u)}\, du + \int_0^{\infty} f(-u+t)\, \overline{\emptyset(-\vartheta u)}\, du \right\} \qquad \textbf{(3)}$$

$$= (W_\emptyset^{+} f)(t,s) + (W_\emptyset^{-} f)(t,s), \qquad \textbf{(4)}$$

where, $\vartheta = \frac{1}{s}$ and t is assumed to be a fixed real number. Setting $f(u+t) = \chi(u)$ and assume that $\chi(u)$ and $\overline{\emptyset(u)}$ are locally integrable on $(0, \infty)$. Further assume that asymptotic expansions of the form [1]:

$$\overline{\emptyset(u)} = \sum_{i=0}^{n-1} s_i\, u^{i-p} + \overline{\emptyset_n(u)}, \text{ as } u \to 0^+, \qquad \textbf{(5)}$$

$$\chi(u) = \sum_{i=0}^{n-1} t_i\, u^{-i-q} + \overline{\chi_n(u)}, \text{ as } u \to +\infty. \qquad \textbf{(6)}$$

Also assume that

$\overline{\emptyset(u)}$ = O $(u^{-\tau})$, as $u \to +\infty, \tau \in R$, **(7)**

and $\chi(u)$ = O $(u^{-\rho})$, as $u \to 0^{+}$, $\rho \in R$. **(8)**

With parameters p, q, τ and ρ satisfying the following condition

$p+\tau < 1 < q+\rho, \tau < q$ and $p < \rho$. **(9)**

Let us remind earlier results [(8), (9), (10), of Theorem 1 [1]]. The asymptotic approximation of continuous wavelet transform $(W_\emptyset f)(t,s)$ given by **(4)** is given below by the following three cases of results [6] as:

Case I: When $n = 1,2,3,\dots$ and $m = n + \lfloor p+q \rfloor$ with $p+q \notin Z$, we have

$(W_\emptyset f)(t,s)$ = $\sum_{i=0}^{n-1} t_i \left[M[\overline{\emptyset(u)}; 1-i-q] + (-1)^{-i-q} M[\overline{\emptyset(-u)}; 1-i-q]\right] s^{-i-q+\frac{1}{2}}$ $+\sum_{i=0}^{n-1} s_i \left[M[\chi(u); 1+i-p] +\right.$ $-1-i-p\ M\chi-u;1+i-p \times s-i+p-1/2$

$+$ O $(s^{-n-q+1/2})$. **(10)**

Case II: When $n = 1,2,3,\dots$ and $m = n+p+q-1$ with $p+q \in N$, we have

$(W_\emptyset f)(t,s)$ = $\sum_{i=0}^{p+q-2} s_i \,[M[\chi(u); 1+i-p] + (-1)^{i-p} M[\chi(-u); 1+i-p]]$

$\times\ s^{-i+p-1/2} + \sum_{i=0}^{n-1} s^{-i-q+1/2} \{ \lim_{z\to 0} [t_i\, M[\overline{\emptyset(u)}; z+1-i-q]$

$+ (-1)^{-i-q} M[\overline{\emptyset(-u)}; z+1-i-q] + s_{i+p+q-1} [M[\chi(u); z+i+q]$

$+ (-1)^{i+q-1} M[\chi(-u); z+i+q]]]\}$ + O $(s^{-m+p-1/2} \log(1\backslash s))$. **(11)**

Case III: When $m = 1,2,3,\dots$ and $n = m+1-p-q$ with $1-p-q \in N$, we have

$(W_\emptyset f)(t,s)$ = $\sum_{i=0}^{-p-q} t_i [M[\overline{\emptyset(u)}; 1-i-q] + (-1)^{-i-q} M[\overline{\emptyset(-u)}; 1-i-q]]$ $\times$ $s^{-i-q+1/2}$ + $\sum_{i=0}^{m-1} s^{-i+p-1/2} \{ \lim_{z\to 0}$ $[\, t_{i+1-p-q}\, [M[\overline{\emptyset(u)}; z+p-i]$

$+ (-1)^{-i-1+p} M[\overline{\emptyset(-u)}; z+p-i]] + s_i [M[\chi(u); z+1+i-p]$

$+ (-1)^{i-p} M[\chi(-u); z+1+i-p]]]\}$ + O $(s^{-m+p-1/2} \log(1\backslash s))$. **(12)**

2. Application

In this section, we obtain asymptotic approximation of the Mexican Hat wavelet transform by using aforesaid technique **(10), (11)** and **(12)** when **s**$\to +\infty$.

2.1. Asymptotic Approximation of the Mexican Hat Wavelet Transform

We choose $\emptyset$ to be Mexican Hat wavelet [11] $\emptyset(u) = (1-u^2)\, e^{\frac{-u^2}{2}}$. Since $\emptyset$ is locally integrable on $(0,\infty)$ and has the asymptotic approximation [1]:

$\overline{\emptyset(u)} = 1 - \frac{3u^2}{2} + \frac{5u^4}{8} - \frac{7u^6}{48} + \frac{3u^8}{128} + O(u^9)$, as $u \to 0^{+}$,

with

$\overline{\emptyset(u)} = O(1), u \to +\infty$.

As $\chi(u)$ is locally integrable on $(0, \infty)$ and satisfies **(6)** and **(8)** with parameters $1 < q+\rho$; $q > 0$ and $0 < \rho$. Now by using **(10), (11)** and **(12)** respectively and by means of formula ([8], p.313, (13)), then the asymptotic approximation of the Mexican Hat wavelet transform for large value of dilation parameter when **s**$\to +\infty$ is given as:

Case I: When $m = 9 + \lfloor q \rfloor$ and $q \notin Z$, we get

$(W_\emptyset f)(t,s) = \sum_{i=0}^{8} t_i \left(1+(-1)^{-i-q}\right) \{ 2^{\frac{1}{2}(-1-i-q)}\, (i+q)\, \Gamma\left[\frac{1}{2}(1-i-q)\right]\}\, s^{-i-q+\frac{1}{2}}$

$+ \sum_{i=0}^{m-1} s_i\, [M[\chi(u); 1+i] + (-1)^i M[\chi(-u); 1+i]]\, s^{-i-\frac{1}{2}}$

$+$ O $(s^{-\frac{17}{2}-q})$. **(13)**

Case II: When $m = 8+q$ and $q \in N$, we have

$(W_\emptyset f)(t,s) = \sum_{i=0}^{q-2} s_i\, [M[\chi(u); 1+i] + (-1)^i M[\chi(-u); 1+i]]$

$\times\ s^{-i-1/2} + \sum_{i=0}^{8} s^{-i-q+1/2} \{ \lim_{z\to 0} [t_i (1+(-1)^{-i-q})$

$\times\ \{ 2^{\frac{1}{2}(-1-i-q+z)}\ (i+q-z)\ \Gamma\left[\frac{1}{2}(1-i-q-z)\right]\}]$

$+\ s_{i+q-1}\, [M[\chi(u); z+i+q] + (-1)^{i+q-1} M[\chi(-u); z+i+q]]]\}$

$+$ O $(s^{-m-1/2} \log(1\backslash s))$. **(14)**

Case III: When $m = 8+q$ and $1-q \in N$, we have

$$(W_{\emptyset} f)(t,s) = \sum_{i=0}^{-q} t_i \left(1+(-1)^{-i-q}\right)\{ 2^{\frac{1}{2}(-1-i-q)}(i+q) \ \Gamma[\tfrac{1}{2}(1-i-q)]\} s^{-i-q+\frac{1}{2}}$$
$$+ \sum_{i=0}^{m-1} s^{-i-\frac{1}{2}} \{\lim_{z\to 0} [t_{i+1-q}\left(1+(-1)^{-i-1}\right)\{ 2^{\frac{1}{2}(-1-i-q+z)} (i+q-z) \ \Gamma[\tfrac{1}{2}(1-i-q+z)] + s_i [M[\chi(u); z+i+1] + (-1)^i M[\chi(-u); z+i+1]]]\}$$
$$+ O(s^{-m-1/2} \log(1\backslash s)). \quad \textbf{(15)}$$

3. Advantages of the Mexican Hat Wavelet Transform

The Mexican Hat Wavelet [12] is derived from the heat kernel by taking the negative first order derivative with respect to time. As a solution to the heat equation it has a clear initial condition: the Laplace- Beltrami operator. Following a popular methodology in mathematics, we analyze the Mexican Hat wavelet and its transforms from a Fourier perspective. The Mexican Hat wavelet is localized in both space and frequency which enables spaces- frequency analysis of input functions. We defined its continuous and discrete transforms as convolution and discrete transforms as convolutions of bivariate kernels and propose a fast method to compute convolutions by Fourier transform [12]. To broader its application scope, we apply the Mexican Hat wavelet to graphics problems of features detection and geometry processing. It plays a vital role for detection of extragalactic point sources in very important issues within the most general problem of the component separation of micro wave sky [13].

4. Conclusion

By using aforesaid techniques we can easily compute the approximation terms of Mexican Hat wavelet transform with their exact error terms. By applying aforesaid results **(10), (11)** and **(12)** we obtained asymptotic approximation of the Mexican Hat wavelet transform $(W_{\emptyset} f)(t,s)$ when s→ +∞.

Acknowledgments

The author has grateful to Dr. Ashish Pathak, Assistant Professor, Department of Mathematics, Institute of Sciences, Banaras Hindu University (BHU),Varanasi-221005, India and Dr. M.M.Dixit, Associate Professor, Department of Mathematics, North Eastern Regional Institute of Science and Technology, (NERIST), Nirjuli-791109, India for their valuable suggestion and guidance for the improvement of the articles. The author also wish to express deep sense of gratitude to Sadanand Yadav, Ojer Yadav, Vijay Kumar Yadav, Dr. Ajay Kumar Yadav, Isha Yadav, Runni Rai (Yadav), Mehak Yadav and Tanishka Yadav for their helping hand and kind support during writing the articles. My deeply gratitude also goes to Miss Laxmi Rai, Assistant Professor, Head, Department of Commerce, DBC, for her fervent supervision, helpful, proper guidance and valuable suggestion, encouragement and constructive criticism during the period of writing the articles. I also express my deep sense of gratitude to my students shri. Geyon Tayeng, Aiyana Perme, Amum Perme and Jenam Padung for their encouragement, stimulating discussion and spent their valuable time with me during the course of writing the articles. I would like to take this opportunity to express my deep sense of regards and thankfulness to Miss Maloty Moyong (Head Teacher) for providing me all sorts of facilities and for their helping hand during the course of writing the articles. Lastly i acknowledge my deep sense of appreciation to all the teaching and non teaching staff of KGBV Rayang who directly or indirectly helped me to complete the articles with their act of assistant and cooperation.

References

1. Pathak,A., Yadav,P., Dixit,M.M., Asymptotic Expansion of Wavelet Transform, Advances in Pure Mathematics, Scientific Research Publishing, vol.5, 21-26, (2015).
2. Pathak,A., Yadav,P., Dixit,M.M**.,** An Asymptotic Expansion of Continuous Wavelet Transform for Large Values of Dilation Parameter, Boletim da Sociedade Paranaense de Matematica, vol., 36 (3)., 27-39, (2018), 2175-1188(online)/ 00378712 IN PRESS.
3. Pathak,A., Yadav, P., and Dixit, M.M., On Convolution for General Novel Fractional Wavelet Transform, Journal of Advanced Research in Scientific Computing, vol.7., Issue.1, 30-37, 2015, Online ISSN:1943-2364.
4. Pathak,A., Yadav,P., Dixit,M.M., An Asymptotic Expansion of Continuous Wavelet Transform for Large Values of b; Investigation in Mathematical Sciences; vol.5, 87-92; (2016), 2250-1436.
5. Yadav,P., A Brief Description of Wavelet and Wavelet Transforms and their Applications, International Journal of Statistics and Applied Mathematics; 3(1): 266-271; (2018); ISSN: 2456-1452.
6. Yadav,P., Extension of Asymptotic Approximations of Continuous Wavelet Transform, International Journal of Statistics and Applied Mathematics; 3(2): 107-110; (2018); ISSN: 2456-1452.
7. Yadav,P., Asymptotic Approximation of the Shannon Wavelet Transform For Large Value of Dilation Parameter, International Journal of Advanced Research and Development, 3(2): 513-517;(2018);ISSN: 2455-4030.
8. Erde'lyi,A., Magnus,W., Oberhettinger,F., and Tricomi,F.G., Tables of Integral Transforms, Vol.1. McGraw-Hill, New York, (1954).
9. Jose Lopez, L., Asymptotic Expansions of Mellin Convolutions by Means of Analytic Continuation

,Journal of Computational and Applied Mathematics, 200: 628-636, (2007).

10. Jos. Lopez,L., and Pedro P., Asymptotic Expansions of Mellin Convolution Integrals: An Oscillatory Case, Journal of Computational and Applied Mathematics, 233, 1562-1569, (2010)..
11. Debnath,L., Wavelet Transforms and their Applications, Birkhauser,(2002).
12. Hou*, T; Qin,H; Corrigendum to “Continuous and Discrete Mexican Hat Wavelet Transforms on Manifolds [Graphical Models 74(2012), 221-232].
13. Gonzalez-Nuevo,J., Argueso.F., Lopez-Caniego.M., Toffolatti,L., Sanz,L,J., Vielva.P., Herranz,D., The Mexican Hat Wavelet Family: Application to Point source detection in Cosmic Microwaves Background Maps., Monthly Notices of the Royal Astronomical society, Volume 369, Issue 4, 11 July 2006, Pages 1603-1610, https:// doi.org/ 10.1111/ j.1365-2966.2006.10442.x.

