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Solution of Non-homogeneous bulk arrival Two-node Tandem Queuing Model using Intervention Poisson distribution

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ABSTRACT

Tandem queuing model is useful for analyzing the situations arising at banks, temples, hospitals, cargo handling, etc. However, in several communication networks, the output of one service station is an input to the other service station. That is, the service stations (transmitters) are connected in tandem. Tandem queuing models provide the basic framework for designing and monitoring of several communication systems. In this paper, it is considered and studied the two-node tandem queuing model (TNTQM) with non-homogeneous compound Poisson bulk arrivals has studied using Intervention Poisson distribution (IPD). By applying sensitivity analysis, the system performance measures are highly influenced by non-homogenous bulk arrivals rate and batch size distribution parameters. Simulation studies showed that the intervened parameter in increasing the service rate in terms of utility, mean number of customer who undertakes service and emptiness pattern in the queue.

1. Introduction

The quality of service (QoS) routing in wireless network which has the multiple hops dealt with two-node tandem queuing model (TNTQM) [5, 8]. An approximate analysis of open systems of tandem queues with blocking caused by finite buffers between servers proposed by [3]. For blocking and failures, [9] has considered the TNTQ system with Bernoulli arrivals and phase type servers, and[11] developed the same having load dependent service rates communication networks, and for bulk arrivals studied by [10]. In communication networks such as Ethernet LAN, MAN traffic, WAN, and VBR traffic exhibit time-dependent arrival rates and cannot be modeled with homogeneous Poisson process or compound Poisson process [2, 4, 16]. The arrival rate of TNTQ of traffic studied by [6]with time varying. Two-node non-homogeneous compound Poisson process depends on the arrivals of the customer in bulky [1].Using tandem queue, limited work has reported in the literature, hence in this paper, we developed and analyzed the TNTQM using IPD and also assumed that the arrivals are time dependent. The joint probability generating function, mean arrivals rate, service rate, mean waiting time, throughput has been derived using difference differential equations for queue and service as well. One special case namely two-node tandem queuing model with non-homogeneous Intervened Poisson bulk arrivals having state dependent service rates are also studied explicitly. The notations used in this paper as t : Time of arrivals; θ: Incidence parameter; ρ: Intervened parameter; λ: Arrival rate; α: Arrivals of infinite population; μ1: First Service rate; μ2: Second Service rate.

2. Queuing Model

The considered queuing model has two queues Q_1, Q_2, service stations S_1, S_2 are connected in series, and the customers will get service through first server then will join into the second queue which is connected to S_1in series. Further the number customers arrived into first queue in batches (the actual number of customers in any arriving module is a random variable X with probability C_k). the arrivals follow compound Poisson process with mean arrival rate λ(t) with bulk size distributions $\{C_k\}$ and the service stations follow the same processes with parameters μ_1 and μ_2 for the first and second service stations which are linearly dependent on the content of queue has the FIFO discipline. The diagram of tandem queue shown in Fig.1

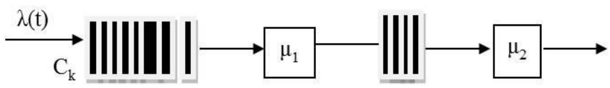

Fig.1. Schematic diagram of the proposed model

Let n_1 and $n_2$be the number of customers of first and second queues and let $P_{n_1,n_2}(t)$ be the probability that there are n_1 customers in the first queue and n_2 customers in the second queue at time t. In the difference-differential equations governing the model, a, b and c are arbitrary constants derived using the initial conditions $P_{0,0}(0)=1, P_{0,0}(t)=0 \text{ for } t>0$

2.1 *Intervened Poisson distribution (IPD)*

The intervened Poisson Distribution (IPD) is an alternate form of zero-truncated Poisson distribution with some interventions [14, 15] and it has wide applications including reliability, medical and etc. The probability mass function of IPD as

$$P(X=x)=\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^{x}-\rho^{x}\right]\right]\theta^{x}}{x!},\theta>0$$

The mean and variance of IPD are

$$E(X)=\left[\theta\left[\rho+1+\left(e^{\theta}-1\right)^{-1}\right]\right]$$ and

$$V(X)=\mu-e^{\theta}\left(\frac{\theta}{(e^{\theta}-1)^{2}}\right)^{2}$$

2.2 Performance Measures with IPD

The number of customers in both queues, the joint probability generating function is given as

$$P(Z_1,Z_2,t)=\exp\left[\sum_{k=1}^{K}\sum_{r=0}^{k}\sum_{i=0}^{r}(-1)^i\left[\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^k-\rho^k\right]\right]\theta^k}{k!}\right]\binom{k}{r}\binom{r}{i}\left[\frac{(z_2-1)\mu_1}{\mu_2-\mu_1}\right]^i\left[(z_2-1)+\left[\frac{(z_2-1)\mu_1}{\mu_2-\mu_1}\right]^{\gamma-i}\right]\left[\frac{1-e^{-\left[i_\mu+(r-i)_\mu\right]^t}}{i\mu_2+(r-i)\mu_1}\right]\right]\right]\lambda+$$

$$\exp\left[\sum_{k=1}^{m}\sum_{r=0}^{k}\sum_{i=0}^{r}(-1)^i\left[\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^k-\rho^k\right]\right]\theta^k}{k!}\right]\binom{k}{r}\binom{r}{i}\left[\frac{(z_2-1)\mu_1}{\mu_2-\mu_1}\right]^i\left[(z_2-1)+\left[\frac{(z_2-1)\mu_1}{\mu_2-\mu_1}\right]^{r-i}\right]\left[\frac{\left[i\mu_2+(r-i)\mu_1\right]t-1-e^{-\left[i_{\mu_2}+(r-i)\mu\right]^t}}{\left[i\mu_2+(r-i)\mu_1\right]^2}\right]\right]\right]\alpha$$

Expanding $P(Z_1,Z_2,t)$ we obtain the probability of zero customers in the system as

$$P_{00}(t)=\exp\left\{\left[\lambda\sum_{k=1}^{K}\sum_{r=1}^{k}\sum_{i=0}^{r}(-1)^{i+r}\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^k-\rho^k\right]\right]\theta^k}{k!}\right]\binom{k}{r}\binom{r}{i}\left[\frac{\mu_2^{r-i}\mu_1^i}{(\mu_2-\mu_1)^r}\right]\left[\frac{1-e^{-\left[i_{\mu_2}+(r-i)\mu\right]^t}}{i\mu_2+(r-i)\mu_1}\right]\right]\right\}+$$

$$\exp\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{i=0}^{r}(-1)^{i+r}\left[\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^k-\rho^k\right]\right]\theta^k}{k!}\right]\binom{k}{r}\binom{r}{i}\left[\frac{\mu_2^{\gamma-i}\mu_1^i}{(\mu_2-\mu_1)^r}\right]\left[\frac{\left[i\mu_2+(r-i)\mu_1\right]t-1-e^{-\left[i_{\mu_2}+(r-i)\mu\right]^t}}{\left[i\mu_2+(r-i)\mu_1\right]^2}\right]\right]\right]$$

The generating function of first queue as

$$P(Z_1,t)=\exp\left[\lambda\sum_{k=1}^{m}\sum_{r=0}^{k}\left[\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^k-\rho^k\right]\right]\theta^k}{k!}\right]\binom{k}{r}(z_2-1)^r\left[\frac{1-e^{-r\mu t}}{r\mu_1}\right]\right]\right.$$

$$\left.+\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\left[\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^k-\rho^k\right]\right]\theta^k}{k!}\right]\binom{k}{r}(z_2-1)^r\left[\frac{tr\mu_1-1-e^{-r\mu t}}{(r\mu_1)^2}\right]\right]\right]$$

By expanding P(Z1,t) we obtain the probability of zero customers in the first queue system as

$$P_0(t)=\exp\left[\lambda\sum_{k=1}^{m}\sum_{r=0}^{r}\left[\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^k-\rho^k\right]\right]\theta^k}{k!}\right]\binom{k}{r}(-1)^r\left[\frac{1-e^{-r\mu t}}{r\mu_1}\right]\right]\right.$$

$$\left.+\alpha\sum_{K=1}^{m}\sum_{r=1}^{K}\left[\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^k-\rho^k\right]\right]\theta^k}{k!}\right]\binom{k}{r}(-1)^r\left[\frac{tr\mu_1-1-e^{-r\mu t}}{(r\mu_1)^2}\right]\right]\right]$$

The average number of customers in the first queue is

$$L_1(t)=\frac{\lambda}{\mu_1}\left[1-e^{-r\mu t}\right]+\frac{\alpha}{(\mu_1)^2}\left[t\mu_1-1-e^{-\mu t}\right]\left[\theta\left[\rho+1+\left(e^{\theta}-1\right)^{-1}\right]\right]$$

The utilization factor of the first service station is $U_1(t)=1-P_{0.}(t)$

$$=1-\exp\left[\lambda\sum_{k=1}^{m}\sum_{r=0}^{k}\left[\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^k-\rho^k\right]\right]\theta^k}{k!}\right]\binom{k}{r}(-1)^{\gamma}\left[\frac{1-e^{-r\mu t}}{r\mu_1}\right]\right]+\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\left[\left[\frac{\left(e^{\theta}-1\right)^{-1}\theta^k}{k!}\right]\binom{k}{r}(-1)r\left[\frac{tr\mu_1-1-e^{-r\mu t}}{(r\mu_1)^2}\right]\right]\right]$$

The throughput of the first service station is $Thp_1(t)=\mu_1U_1(t)$

$$=\mu_1\left[1-\exp\left[\lambda\sum_{k=1}^{m}\sum_{r=0}^{k}\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^k-\rho^k\right]\right]\theta^k}{k!}\right]\binom{k}{r}(-1)^r\left[\frac{1-e^{-r\mu t}}{r\mu_1}\right]\right]\right.$$

$$=\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\left[\left[\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^{k}-\rho^{k}\right]\right]\theta^{k}}{k!}\right]\binom{k}{r}(-1)^{r}\left[\frac{tr\mu_1-1-e^{-r\mu t}}{(r\mu_1)^2}\right]\right]\right]$$

The mean waiting time, variance and coefficient of variation of a customer in the first queue as

$$W_1(t)=\frac{L_1(t)}{Thp_1(t)}$$

$$V_1(t)=\sum_{k=1}^{m}\left[\left[k\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^{k}-\rho^{k}\right]\right]\theta^{k}}{k!}\right]\frac{\lambda}{\mu_1}\left[\frac{k-1}{2}\right]\left[1-e^{-2\mu t}\right]+\left[1-e^{-\mu t}\right]\right]\right.$$

$$\left.+\frac{\alpha}{(\mu_1)^2}\left[\left[\frac{k-1}{2}\right]\left[\frac{2t\mu_1-1+e^{-2\mu t}}{(r\mu_1)^2}\right]+\left[t\mu_1-1-e^{-\mu t}\right]\right]\right]$$

$$CV_1(t)=\frac{\sqrt{V_1(t)}}{L_1(t)}\times 100$$

Again expanding the $P(Z_1,Z_2,t)$, we obtain the probability of zero customers in the system as

$$P_{0.}(t)=\exp\left\{\left[\lambda\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{i=0}^{r}(-1)^{i+r}\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^{k}-\rho^{k}\right]\right]\theta^{k}}{k!}\right]\binom{k}{r}\binom{r}{i}\left[\frac{\mu_1}{(\mu_2-\mu_1)}\right]^{r}\left[\frac{1-e^{-\left[i_{\mu_2}+(r-i)\mu\right]^{t}}}{i\mu_2+(r-i)\mu_1}\right]\right]\right\}+$$

$$\exp\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{i=0}^{r}(-1)^{i+r}\left[\left[\frac{\left(e^{\theta}-1\right)^{-1}\theta^{k}}{k!}\right]\binom{k}{r}\binom{r}{i}\left[\frac{\mu_1}{(\mu_2-\mu_1)}\right]^{r}\left[\frac{\left[i\mu_2+(r-i)\mu_1\right]t-1-e^{-\left[i_{\mu_2}+(r-i)\mu\right]^{t}}}{\left[i\mu_2+(r-i)\mu_1\right]^{2}}\right]\right]\right]$$

The average number of customers in the second queue is

$$L_2(t)=\left(\frac{\mu_1}{\mu_2-\mu_1}\right)\frac{\lambda}{\mu_1}\left[1-e^{-(\mu_1 t)}\right]+\frac{\alpha}{(\mu_1)^2}\left[t\mu_1-1-e^{-\mu_1 t}\right]\left[\theta\left[\rho+1+\left(e^{\theta}-1\right)^{-1}\right]\right]$$

The utilization factor of the second service station is $U_2(t)=1-P_{.0}(t)$

$$=1-\exp\left\{\left[\lambda\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{i=0}^{r}(-1)^{i+r}\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^{k}-\rho^{k}\right]\right]\theta^{k}}{k!}\right]\binom{k}{r}\binom{r}{i}\left[\frac{\mu_1}{(\mu_2-\mu_1)}\right]^{r}\left[\frac{1-e^{-\left[i_{\mu_2}+(r-i)\mu\right]^{t}}}{i\mu_2+(r-i)\mu_1}\right]\right]\right\}+$$

$$\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{i=0}^{r}(-1)^{i+r}\left[\left[\frac{\left(e^{\theta}-1\right)^{-1}\theta^{k}}{k!}\right]\binom{k}{r}\binom{r}{i}\left[\frac{\mu_1}{(\mu_2-\mu_1)}\right]^{r}\left[\frac{\left[i\mu_2+(r-i)\mu_1\right]t-1-e^{-\left[i_{\mu_2}+(r-i)\mu\right]^{t}}}{\left[i\mu_2+(r-i)\mu_1\right]^{2}}\right]\right]\right]$$

The throughput of the second service station is $Thp_2(t)=\mu_1 U_2(t)$

$$=\mu_1\left[1-\exp\left\{\left[\lambda\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{i=0}^{r}(-1)^{i+r}\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^{k}-\rho^{k}\right]\right]\theta^{k}}{k!}\right]\binom{k}{r}\binom{r}{i}\left[\frac{\mu_1}{(\mu_2-\mu_1)}\right]^{r}\left[\frac{1-e^{-\left[i_{\mu_2}+(r-i)\mu\right]^{t}}}{i\mu_2+(r-i)\mu_1}\right]\right]\right\}+\right.$$

$$\left[\alpha\sum_{k=1}^{m}\sum_{r=1}^{k}\sum_{i=0}^{r}(-1)^{i+r}\left[\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^{k}-\rho^{k}\right]\right]\theta^{k}}{k!}\right]\binom{k}{r}\binom{r}{i}\left[\frac{\mu_1}{(\mu_2-\mu_1)}\right]^{r}\left[\frac{\left[i\mu_2+(r-i)\mu_1\right]t-1-e^{-\left[i_{\mu_2}+(r-i)\mu\right]^{t}}}{\left[i\mu_2+(r-i)\mu_1\right]^{2}}\right]\right]\right]$$

The mean waiting time, variance and coefficient of variation of a customer in the first queue as

$$W_2(t)=\frac{L_2(t)}{Thp_2(t)}$$

$$V_2(t)=\sum_{k=2}^{m}\left[\left[(k).\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^{k}-\rho^{k}\right]\right]\theta^{k}}{k!}\right].\left(\frac{k-1}{2}\right).\left(\frac{\mu_1}{\mu_2-\mu_1}\right)^{2}\right]\left[\lambda\left[\frac{\left[1-e^{-(2\mu_1 t)}\right]}{\mu_1}-4\left[\left(\frac{\mu_1}{\mu_2-\mu_1}\right)^{2}\right]\frac{\left[1-e^{-(2\mu_1 t)}\right]}{\mu_2}\right]+\alpha\left[\frac{(2t\mu_1)-1+e^{-(2\mu_1 t)}}{2(\mu_1)^2}\right]\right.$$

$$\left.-4\left[\left[\frac{t(\mu_1+\mu_2)-1+e^{-(\mu_1+\mu_2)t}}{(\mu_2+\mu_1)^2}\right]^{2}+\left[\frac{(2t\mu_2)-1+e^{-(2\mu_2 t)}}{2(\mu_2)^2}\right]\right]\right]$$

$$+\sum_{k=2}^{m}\left[\left[(k)\left[\frac{\left[e^{\rho\theta}\left(e^{\theta}-1\right)^{-1}\left[(1+\rho)^{k}-\rho^{k}\right]\right]\theta^{k}}{k!}\right]\cdot\left(\frac{\mu_1}{\mu_2-\mu_1}\right)^{2}\right]\left[\left(\frac{\lambda}{\mu_1}\right)\left[1-e^{-(\mu_1 t)}\right]+\frac{\alpha}{(\mu_1)^{2}}+\left[t\mu_1-1+e^{-(\mu_1 t)}\right]\right]\right]$$

$$CV_2(t)=\frac{\sqrt{V_2(t)}}{L_2(t)}\times 100$$

The mean number of customers in entire system at time t is $L(t)=L_1(t)+L_2(t)$

3. An illustration

Through the numerical example, we discussed the performance measures for the proposed model. As discussed in the second section, the number of customers enters into the queue system in bulky way, once the service taken from first queue, then will join into the second which are serially connected. The arrival arte and module of the customers follows compound Poisson process and Uniform distribution respectively. Using this illustration, the performance measures were computed in transient form, and the parameters considered as:

t : 0.05, 0.06, 0.07, 0.08, 0.09;
θ : 1.50, 2.0, 2.5 ,3, 3.5;
ρ: 1.50, 2.0 ,2.5, 3, 3.5;
λ: 0.5, 0.6, 0.7, 0.8, 0.9;
α: 0.02, 0.03, 0.04, 0.05, 0.06;
μ_1: 19, 20, 21, 22, 23;
μ_2: 26,27,28,29,30,31;

The probability of zero customers in the queuing system and marginal queues, the average number of customers, utilization factor were computed for different values of the parameters t, θ, ρ, λ, α, μ_1, μ_2, and were presented in Table. 1 and throughput of the service stations, mean waiting time, variance and coefficient of variation of customers in each queue were given in Table.2. The relationships between the parameters and performance measures are shown in Fig.2. The similar graphical representation is also made for the performance measures with respect to λ, μ_1 and μ_2 respectively. From all the comparisons, it is observed in the Table. 3

4. Conclusion

In this paper the applicability of the intervened Poisson distribution is made to demonstrate the behavior of non-homogeneous bulk queues under two node tandem server setup upon the simulation studies it is observed that the intervened parameter in increasing the service rate in terms of utility, mean number of customer who under take service and emptiness pattern in the queue. However due to the presence of compound parameter the waiting time is observed to have a gradual increasing pattern but this can be minimized if the service rate is increased on the whole the imparting IPD in to prime work of Non-homogeneous bulk arrivals as shown its impact in explained behavior of the queuing system further controlling they parameters like variability, increase the service rate the model provides better result in terms of throughput, utility and waiting time.

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Appendix

Table.1.

Values of $P_{0.}(t)$, $P_{.0}(t)$, $P_{00}(t)$, $L_1(t)$, $L_2(t)$, $L(t)$, $U_1(t)$, $U_2(t)$ for different values of parameters

t	θ	ρ	λ	α	μ_1	μ_2	$P_{0.}(t)$	$P_{00}(t)$	$P_{.0}(t)$	$L_1(t)$	$L_2(t)$	L (t)	$U_1(t)$	$U_2(t)$
0.05	1		0.4	0.01	18	26	0.983	0.981	0.991	0.034	0.03	0.064	0.017	0.009
0.06							0.98	0.977	0.989	0.038	0.033	0.071	0.02	0.011
0.07							0.978	0.974	0.987	0.041	0.036	0.077	0.022	0.013
0.08							0.976	0.971	0.985	0.044	0.038	0.0832	0.024	0.015
0.09							0.974	0.968	0.983	0.046	0.040	0.086	0.026	0.017
	1.5						0.9819	0.9805	0.99	0.045	0.0297	0.075	0.0180	0.01
	2						0.9813	0.9803	0.988	0.057	0.0298	0.087	0.0186	0.012
	2.5						0.9809	0.9802	0.987	0.069	0.0298	0.099	0.0191	0.013
	3						0.9806	0.9801	0.986	0.081	0.0298	0.111	0.0193	0.014
	3.5						0.9804	0.9800	0.985	0.094	0.0298	0.124	0.0195	0.015
		1.5					0.9823	0.9806	0.99016	0.041	0.02974	0.070	0.0177	0.010
		2					0.9818	0.9805	0.98923	0.047	0.02975	0.077	0.0182	0.011
		2.5					0.9814	0.9804	0.98841	0.054	0.02976	0.084	0.0186	0.012
		3					0.9811	0.9803	0.98768	0.060	0.02977	0.090	0.0189	0.013
		3.5					0.9809	0.9802	0.98703	0.067	0.02978	0.097	0.0191	0.014
			0.5				0.9788	0.9761	0.9890	0.043	0.0371	0.080	0.0212	0.011
			0.6				0.9746	0.9714	0.9868	0.051	0.0446	0.096	0.0254	0.013
			0.7				0.9704	0.9667	0.9847	0.060	0.0520	0.112	0.0296	0.015
			0.8				0.9662	0.9621	0.9825	0.068	0.0594	0.128	0.0338	0.017
			0.9				0.9621	0.9574	0.9803	0.077	0.0668	0.143	0.0379	0.020
				0.02			0.983	0.9808	0.9912	0.0339	0.0298	0.06388	0.01704	0.0088
				0.03			0.983	0.9808	0.9912	0.0340	0.0298	0.06396	0.01705	0.0088
				0.04			0.983	0.9808	0.9912	0.0341	0.0298	0.06404	0.01706	0.0088
				0.05			0.983	0.9808	0.9912	0.0342	0.0298	0.06412	0.01707	0.0088
				0.06			0.983	0.9808	0.9912	0.0343	0.0298	0.0642	0.01707	0.0088
					19		0.9832	0.98087	0.991	0.0334	0.0351	0.0685	0.0168	0.009
					20		0.9834	0.98091	0.9907	0.0327	0.0422	0.0749	0.0166	0.0093
					21		0.9835	0.98095	0.9905	0.0320	0.0521	0.0841	0.0165	0.0095
					22		0.9837	0.98098	0.9903	0.0313	0.0668	0.0982	0.0163	0.0097
					23		0.9839	0.98102	0.9901	0.0307	0.0913	0.122	0.0164	0.0099
						27	0.983	0.9809	0.9913	0.0341	0.0264	0.0605	0.017	0.0087
						28	0.983	0.9809	0.9914	0.0341	0.0238	0.0579	0.017	0.0086
						29	0.983	0.9809	0.9915	0.0341	0.0216	0.0557	0.017	0.0085
						30	0.983	0.9809	0.9916	0.0341	0.0198	0.0539	0.017	0.0084
						31	0.983	0.9809	0.9916	0.0341	0.0183	0.0524	0.017	0.0084

Table.2.
Values of $Thp_1(t)$, $Thp_2(t)$, $W_1(t)$, $W_2(t)$, $V_1(t)$, $V_2(t)$, $CV_1(t)$, $CV_2(t)$ for different values of parameter

t	θ	ρ	λ	α	μ_1	μ_2	$Thp_1(t)$	$Thp_2(t)$	$W_1(t)$	$W_2(t)$	$V_1(t)$	$V_2(t)$	$CV_1(t)$	$CV_2(t)$
0.05	1	1	0.4	0.01	18	26	0.306	0.228	0.111	0.13	0.085	7.638	853.526	9297.26
0.06							0.352	0.286	0.108	0.116	0.091	8.514	796.794	8816.462
0.07							0.393	0.342	0.105	0.105	0.097	9.246	755.038	8466.454
0.08							0.429	0.394	0.102	0.097	0.101	9.858	723.345	8203.066
0.09							0.46	0.444	0.1	0.091	0.104	10.369	698.759	80000.028
	1.5						0.325	0.272	0.139	0.109	0.146	10.142	842.603	10706.444
	2						0.336	0.31	0.169	0.096	0.222	12.74	828.364	11992.103
	2.5						0.344	0.34	0.201	0.087	0.316	15.417	815.861	13183.646
	3						0.349	0.366	0.233	0.081	0.428	18.158	805.545	14298.255
	3.5						0.352	0.387	0.267	0.077	0.559	20.949	797.095	15347.175
		1.5					0.318	0.256	0.128	0.116	0.119	9.113	846.65	10151.733
		2					0.328	0.280	0.144	0.106	0.157	10.587	837.67	10937.961
		2.5					0.334	0.301	0.161	0.099	0.199	12.060	828.843	11669.751
		3					0.340	0.320	0.178	0.093	0.246	13.531	820.93	1236.78
		3.5					0.343	0.337	0.195	0.088	0.298	15.002	813.977	13006.146
			0.5				0.382	0.285	0.1114	0.1302	0.106	9.55	763.468	8319.788
			0.6				0.458	0.342	0.1117	0.1303	0.127	11.462	696.978	7597.368
			0.7				0.533	0.398	0.1119	0.1304	0.148	13.374	645.297	7035.434
			0.8				0.608	0.455	0.1121	0.1306	0.169	15.286	603.634	6582.195
			0.9				0.682	0.511	0.1124	0.1307	0.190	17.198	569.122	3206.596
				0.02			0.3067	0.2286	0.111	0.1303	0.0846	7.629	853.244	9247.582
				0.03			0.3069	0.2287	0.111	0.1304	0.0847	7.620	852.961	9251.984
				0.04			0.3071	0.2289	0.111	0.1306	0.0848	7.611	852.679	9229.467
				0.05			0.3073	0.2290	0.111	0.1307	0.0849	7.602	852.397	9207.028
				0.06			0.3075	0.2291	0.111	0.1308	0.0850	7.593	852.115	9184.669
					19		0.320	0.2352	0.104	0.1493	0.082	9.022	858.778	8555.446
					20		0.333	0.2415	0.098	0.1748	0.080	10.851	864.132	7801.938
					21		0.346	0.2475	0.093	0.2105	0.077	13.391	869.585	7023.243
					22		0.358	0.2532	0.088	0.2640	0.075	17.178	875.133	6200.86
					23		0.370	0.2586	0.083	0.353	0.073	23.461	880.775	5305.822
						27	0.306	0.2349	0.111	0.1125	0.085	6.792	853.526	9862.642
						28	0.306	0.2412	0.111	0.0986	0.085	6.114	853.526	10397.486
						29	0.306	0.2473	0.111	0.0874	0.085	5.559	853.526	10906.261
						30	0.306	0.2533	0.111	0.0782	0.085	5.097	853.526	11392.443
						31	0.306	0.2592	0.111	0.0706	0.085	4.706	4.706	11858.794

Table.3.
Comparisons

	Emptiness			Mean time of customers			Variance		Utilization		Throughput		Waiting time	
	$P_0(t)$	$P_{.0}(t)$	$P_{00}(t)$	$L_1(t)$	$L_2(t)$	$L(t)$	$V_1(t)$	$V_2(t)$	$U_1(t)$	$U_2(t)$	$Thp_1(t)$	$Thp_2(t)$	$W_1(t)$	$W_2(t)$
Time	Decrease	Decrease	Decrease	Increase	Increase	Increase	Stable	Increase	Increase	Increase	Increase	Increase	Decrease	Decrease
Θ	Decrease	Decrease	Stable	Increase	Stable	Increase	Stable	Increase	Increase	Increase	Increase	Increase	Increase	Decrease
P	Decrease	Decrease	Decrease	Increase	Increase	Increase	Stable	Increase	Increase	Increase	Increase	Increase	Increase	Decrease
Λ	Decrease	Decrease	Decrease	Increase	Increase	Increase	Stable	Increase	Increase	Increase	Increase	Increase	Stable	Stable
A	Stable	Stable	Stable	Stable	Stable	Stable	Stable	Increase	Stable	Stable	Stable	Stable	Stable	Stable
μ_1	Decrease	Stable	Stable	Decrease	Increase	Increase	Stable	Increase	Increase	Increase	Increase	Increase	Decrease	Increase
μ_2	Decrease	Stable	Stable	Decrease	Decrease	Decrease	Stable	Decrease	Stable	Stable	Increase	Increase	Increase	Decrease
	Table. 3. Comparisons													

Fig. 1.
Performance Measures with respect to Time 't'