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# Forbidden induced subgraphs of the Line graph and Covering Preserving Subposets

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**ABSTRACT**

The cover-incomparability graph of a poset P is the edge-union of the covering and the incomparability graph of P. We characterize some ⊲ - preserving subposets of the posets whose cover-incomparability graph contains forbidden induced subgraph of the line graph.

## 1. Introduction

Cover-incomparability graphs of posets, or shortly C-I graphs, were introduced in [2] as the underlying graphs of the standard interval function or transit function on posets (for more on transit functions in discrete structures [3, 4, 5, 6, 11]). On the other hand, C-I graphs can be defined as the edge-union of the covering and incomparability graph of a poset; in fact, they present the only non-trivial way to obtain an associated graph as unions and/or intersections of the edge sets of the three standard associated graphs (i.e. covering, comparability and incomparability graph). In the paper that followed [9], it was shown that the complexity of recognizing whether a given graph is the C-I graph of some poset is in general NP-complete. In [1] the problem was investigated for the classes of split graphs and block graphs, and the C-I graphs within these two classes of graphs were characterized. This resulted in a linear-time recognition algorithms for C-I block and C-I split graphs. It was also shown in [1] that whenever a C-I graph is a chordal graph, it is necessarily an interval graph, however a structural characterization of C-I interval graphs (and thus C-I chordal graphs) is still open. C-I distance-hereditary graphs have been characterized and shown to be efficiently recognizable [10].Let P = (V;_≤) be a poset. If u ≤ v but u ≠ v, then we write u < v. For u, v ϵ V we say that v *covers* u in P if u < v and there is no w in V with u <w < v. If u ≤ v we will sometimes say that u is *below* v, and that v is *above* u. Also, we will write u ⊲ v if v covers u; and u ⊲⊲ v if u is below v but not covered by v. By u ‖ v we denote that u and v are incomparable. Let $V'$ be a nonempty subset of V. Then there is a natural poset Q =($V'$;≤ '), where u ≤ ' v if and only if u ≤ v for any u,v ϵ$V'$. The poset Q is called a *subposet* of P and its notation is simplified to Q = ($V'$;≤). If, in addition, together with any two comparable elements u and v of Q, a chain of shortest length between u and v of P is also in Q, we say that Q is an isometric subposet of P. Recall that a poset P is *dual* to a poset Q if for any x, y ϵ P the following holds: x ≤ y in P if and only if y ≤ x in Q. Given a poset P, its cover-incomparability graph GP has V as its vertex set, and uv is an edge of $G_P$ if u ⊲ v, v ⊲ u, or u and v are incomparable. A graph that is a cover-incomparability graph of some poset P will be called a C-I graph.

**Lemma 1 [2]** Let P be a poset and $G_P$ its C-I graph. Then
*(i)* $G_P$ is connected;
*(ii)* vertices in an independent set of $G_P$ lie on a common chain of P;
*(iii)* an antichain of P corresponds to a complete subgraph in $G_P$ ;
*(iv)* $G_P$ contains no induced cycles of length greater than 4.

## 2. Two-colored and Three-colored diagrams

2-coloured diagram P; in [12] we describe the family P by the Hasse diagram of initial poset P using normal edges, added by the bold edges between $u_i$ and $v_j$ ($u_i$ and $v_j$ are incomparable pairs) for all i and j. It follows that if there is a bold edge between an incomparable pair of elements $u_i$ and $v_j$ in P then either $u_i$⊲ $v_j$ or $v_j$⊲ $u_i$,which neither affect the covering nor the incomparability relation of any other pair of elements in P. Any subset of the set of bold edges can thus be chosen and removed arbitrarily to obtain one of the Hasse diagram of a poset from the family P. Hence one drawing, using normal and bold edges, suffices to describe all posets of P.

A 3-coloured diagram Q in [13] is explained as follows. Let G be a C-I graph and H be an induced subgraph of G. We note that there can be different ⊲- preserving subposets Qi of some posets with $GQi$ isomorphic to the subgraph H. Let u,v,w be an induced path in the direction from u to v in H. There are four possibilities in which u, v and w can be related in the ⊲ - preserving subposets. It is possible to have u ⊲ v, u ‖ v, v ⊲ w and v ‖ w. Each case will appear as a ⊲ - preserving subposet of four different posets. If u ⊲ v and v ⊲ w in a subposet, then u ⊲ v ⊲ w is a chain in the subposet and u,v,w is an induced path in H. If there is either u **||** v or v **||** w in a subposet Q, then there should be another chain from u to w in Q in order to have u, v,w an induced path in H. We try to capture this situation using the idea of 3-colored diagram. Suppose in $\lhd$ - preserving subposet Q of a poset P, there exists two elements u,v which is always connected by some chain of length three in Q. Let w be an element in Q such that either both uw and vw are red edges or any one of them is a red edge. Then in order to have a chain between u and v, there must exist an element x in Q so that u, x, v form a chain in Q. When both edges are normal, then we have the chain u,w, v in Q and hence the chain u, x, v is not required in this case. We denote the chain u, x, v by dashed lines between ux and xv in order to specify that it is possible to have the presence or absence of the chain u, x, v in Q. The presence of the chain u, x, v implies that either both of the edges uw and wv are red edges or one of them is a red edge. The absence of the chain implies that both uw and vw are normal edges in Q. We call posets having the above mentioned diagrams as 3-colored diagrams.

**Theorem 2:** (Theorem 1,[8]): Let G be a class of graphs with a forbidden induced subgraphs characterization. Let $\mathscr{P}$ = {P | P is a poset with $G_{TP} \in$ G }. Then P has a characterization by forbidden $\lhd$ - preserving subposets.

**Theorem 3:** (Theorem 7.1.8, [7]) Let G be a graph. Then G is a line graph if and only if G contains none of the nine forbidden graphs of Figure 1 as an induced subgraph

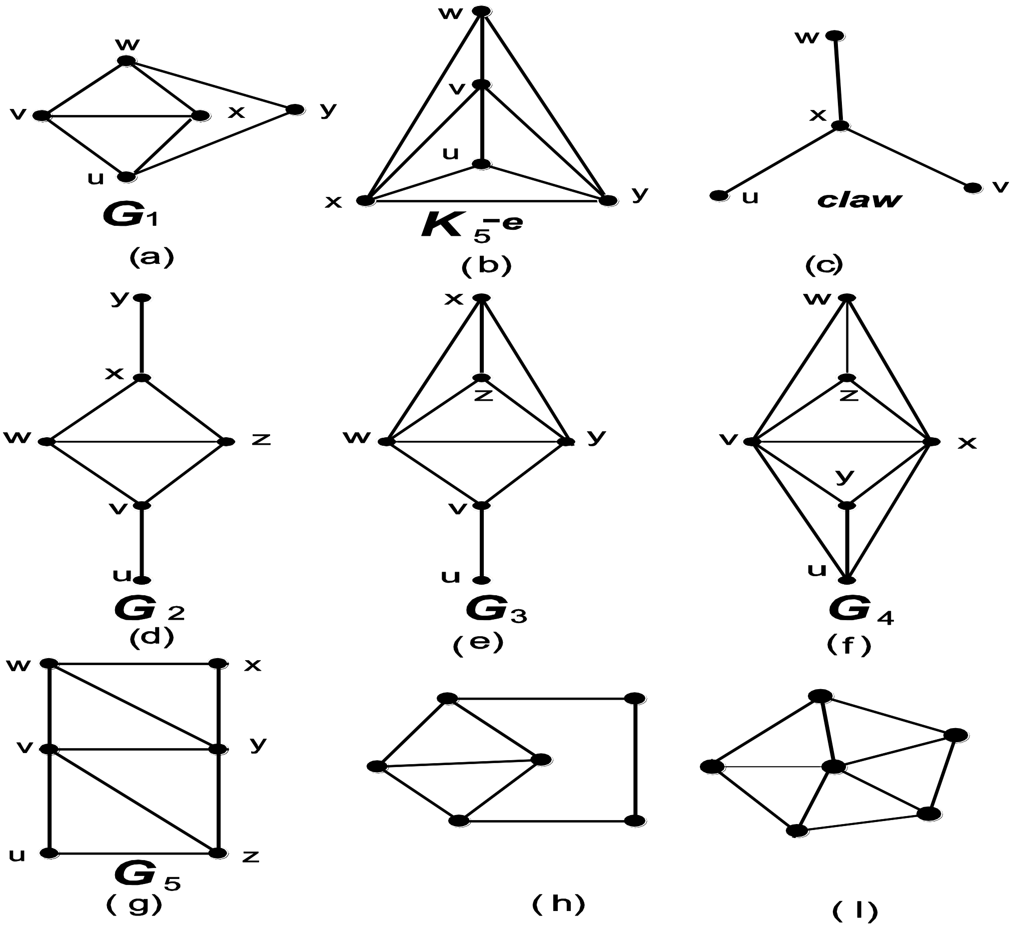



**FIGURE 1:** NINE FORBIDDEN INDUCED SUB GRAPHS OF LINE GRAPH

Let $\mathcal{F}$ (**G**) be a collection of forbidden induced subgraphs. We consider the posets whose cover-incomparability graph **G**$_P$ contains the graphs in $\mathcal{F}$(**G**).

**Theorem 4:** (Theorem 4.1,[12]) If P is a poset, then **GP** contains G1 in $\mathcal{F}$(**G**) of Figure1 if and only if P contains $\lhd$-preserving 2-colored diagrams **P1**, P**2** and **P3** from Figure 2 and their duals.

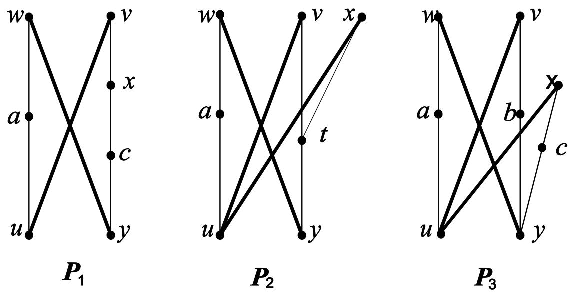
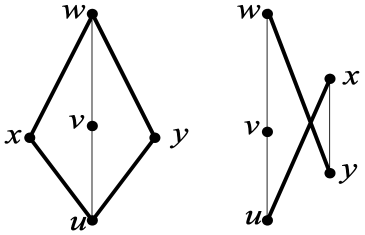



**Figure 2:**Forbidden 2-colored diagrams for posets whose C-I graphs contains G1

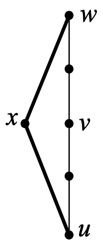
**Theorem 5:** (Theorem 4.2,[12]) If P is a poset, then $\mathbf{G_P}$ contains ($K5 - e$) in $\mathcal{F}$(**G**) of Figure 1 if and only if P contains $\lhd$-preserving 2-colored diagrams $\mathbf{P_4}$ and $\mathbf{P_5}$ from Figure 3 and their duals.




$P_4$ $P_5$

**Figure 3**:Forbidden 2-colored diagrams for posets whose C-I graphs contains ($K5 - e$), depicted in Figure 1 (b)

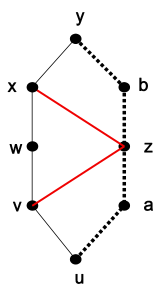
**Theorem 6:** (Theorem 4.3,[12]) If P is a poset, then $\mathbf{G_P}$ contains Claw in $\mathcal{F}$(**G**) of Figure 1 if and only if P contains $\lhd$-preserving 2-colored diagram $\mathbf{P_6}$ from Figure 4 and their duals.




$P_6$

**Figure 4:**Forbidden 2-colored diagrams for posets whose C-I graphs contains Claw, depicted in Figure 1(c)

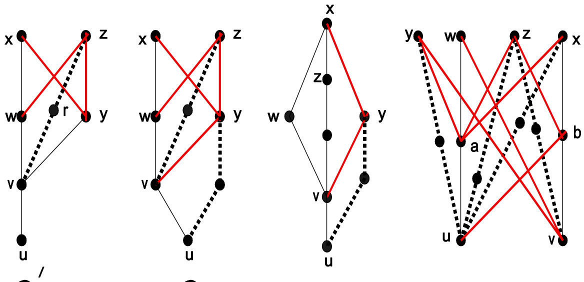
**Theorem 7:** (Theorem 4,[14]) If P is a poset, then $\mathbf{G_P}$ contains $G_2$ in F (**G**) of Figure 1 if and only if P contains $\lhd$-preserving 3-colored diagram $\mathbf{Q_1}$ from Figure 5 and their duals.




$Q_1$

**Figure 5:** Forbidden 3-colored diagrams for posets whose C-I graphs contains $G_2$, depicted in Figure 1 (d).

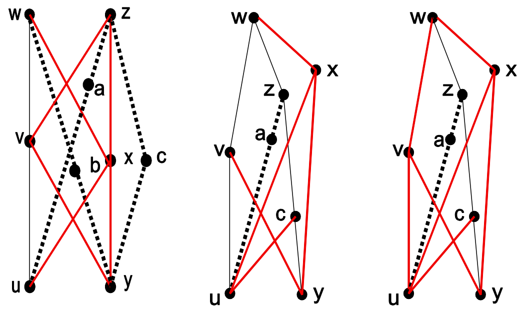
**Theorem 8:** (Theorem 6,[15]) If P is a poset, then $\mathbf{G_P}$ contains $G_3$ in $\mathcal{F}$(**G**) of Figure 1 if and only if P contains $\lhd$-preserving 3-colored diagrams $\mathbf{Q_i}$**, i=2,3,4** from Figure 6 and their duals.




$Q'_2$ $Q_2$ $Q_3$ $Q_4$

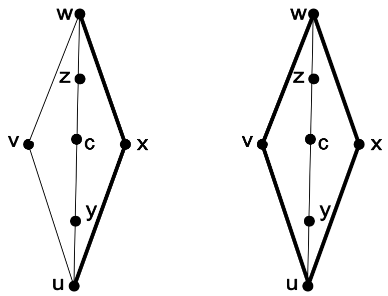
**Figure 6 :** Forbidden 3-colored diagrams for posets whose C-I graphs contains $G_3$, depicted in Figure 1 (e).

**Theorem 9:** (Theorem 5,[16]) If P is a poset, then $\mathbf{G_P}$ contains G4 in $\mathcal{F}$ (**G**) of Figure 1 if and only if P contains the $\lhd$-preserving 3-colored diagrams $Q_5$ and $Q_6$ from Figure 7 and 2-colored diagram $P_7$ from Figure 8 and their duals.




$Q_5$ $Q'_6$ $Q_6$

**Figure 7:** Forbidden 3-colored diagrams for posets whose C-I graphs contains G4, depicted in Figure 1(f)




$P'_7$ $P_7$

**Figure 8:** Forbidden 2-colored diagrams for posets whose C-I graphs contains $G_4$, depicted in Figure 1(f).

**Theorem 10:** (Theorem 5,[17]) If P is a poset, then $\mathbf{G_P}$ contains $G_5$ in $\mathcal{F}$ (**G**) of Figure 1 if and only if P contains the $\lhd$-preserving 3-colored diagrams $Q_7$ and $Q_8$ from Figure 9 , $\lhd$-preserving subposets $U_1$ , $U_2$ from Figure 10 and their duals

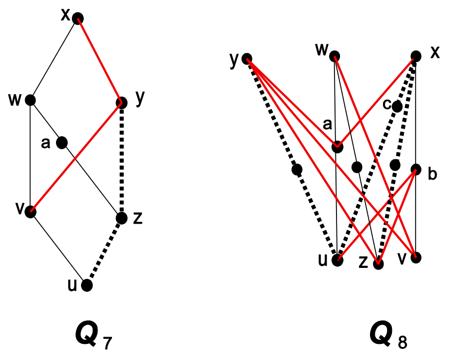



**Figure 9:**Forbidden 3-colored diagrams for posets whose C-I graphs contains $G_5$, depicted in Figure 1 (g).

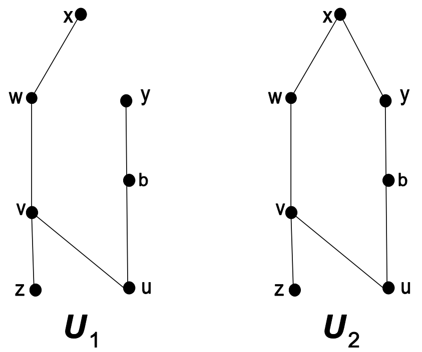



**Figure 10:** Forbidden subposets whose C-I graphs contains $G_5$, depicted in Figure 1(g).

Since (h) and (i) in Figure 1 have induced cycles of length 5,they are not induced subgraphs of Cover-incomparable graphs( by Lemma 3.3,[2] ). Thus we have proved the following theorem.

**Theorem 10:** If P is a poset, then $\mathbf{G_P}$ is a line graph if and only if P has no $\lhd$ - preserving subposets isomorphic to $U_1$ , $U_2$, no 2-colored($\lhd$ - preserving) subposets isomorphic to $P_i$, i=1,2,…,7 and no 3-colored ($\lhd$ - preserving) subposets isomorphic to $Q_i$, i=1,2,…,8

**Remarks :** The number of forbidden $\lhd$ - preserving subposets of a poset P is such that its C-I graph $G_P$ belongs to a graph possessing a forbidden induced subgraph characterization as instances of the Theorem 2 is in general very large compared to the number of forbidden induced subgraphs. Here we characterize $\lhd$ - preserving subposets whose $G_P$ contains forbidden induced subgraphs of line graph in Figure1 and introduce the idea of 2-colored and 3-colored diagrams to minimize the list of subposets.

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