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Surface waves in fiber-reinforced anisotropic elastic media

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1. Introduction

After the pioneer work of Rayleigh (1885), many investigators have been studied the problem extensively under different conditions. They have contributed in a wide range towards its application in various fields i.e. Seismology, geophysics, acoustics, telecommunications and environmental sciences etc.

In last many investigations the reinforcement effect was neglected. Belfield et al. (1983) has introduced this concept. The components of reinforced composite behave like a single anisotropic component till they ate in elastic condition. Both the parts have no relative displacement between them.

Chatopadhyay and Venkateswarlu (1998) studied the stress generated because of moving load in fiber reinforced half space.

Sengupta&Nath (2001) used the method of potential to decouple the P & SV motions for the study of surface waves, but it was not justified. Therefore Singh (2004) pointed out that the results of Sengupta and Nath (2001) are in error regarding Rayleigh and Stoneley waves.

The results are compared with corresponding classical results for isotropic elastic medium in the absence of reinforced elastic parameters.

1.1 Basic Equation

In a Fiber-reinforced linearly elastic anisotropic media Belfield et al. (1983) formulated the below expression for constitutive equations versus reinforcement direction $\vec{a}$

$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu T e_{ij} + \alpha\left(a_k a_m e_{km}\delta_{ij} + e_{kk} a_j a_j\right) + 2(\mu_L - \mu_T)\left(a_i a_k e_{kj} + a_j a_k e_{ki}\right) + \beta a_k a_m e_{km} a_i a_{j,}$..................................... (1.1)

Here the constant and the variable components mentioned in this above equations are as:

1. σ_{ij}stands for stress and
2. e_{ij}stands for strain and
3. λ, μ_T refers to the elastic constants and
4. α, β, (μ_L-μ_T) refers to the reinforcement parameters

$\vec{a} = (a_1, a_2, a_3)$,

$$a_3^2 + a_2^2 + a_3^2 = 1$$

The direction of the Fiber is assumed as $\vec{\mathbf{a}}\,\vec{\mathbf{a}}$ = (1, 0, and 0).

The below expression shows the strains in the form of displacements u_i

$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ . ... (1.2)

Equation (1.1) then yields

$\sigma_{11} = A_{11}\frac{\partial u_1}{\partial x_1} + A_{12}\frac{\partial u_2}{\partial x_2} + A_{13}\frac{\partial u_3}{\partial x_3}$,

$\sigma_{22} = A_{12}\frac{\partial u_1}{\partial x_1} + A_{22}\frac{\partial u_2}{\partial x_2} + A_{23}\frac{\partial u_3}{\partial x_3}$,

$\sigma_{33} = A_{12}\frac{\partial u_1}{\partial x_1} + A_{23}\frac{\partial u_2}{\partial x_2} + A_{22}\frac{\partial u_3}{\partial x_3}$,

$\sigma_{21} = \sigma_{12} = \mu L\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$,

$\sigma_{31} = \sigma_{13} = \mu L\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right)$,

$\sigma_{23} = \sigma_{23} = \mu\mathrm{T}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right)$ .... (1.3)

Here

$$A_{11} = \lambda + 2\alpha + 4\mu_T - 2\mu_T + \beta, \quad A_{12} = \lambda + \alpha,$$

$A_{22} = \lambda + 2\mu_T, A_{23} = \lambda,$ ……............... (1.4)

Neglecting the body forces, the motion's equation can be mentioned as

$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho\frac{\partial^2 u_i}{\partial t^2}$(ij= 1,2,3) , ... (1.5)

Here ρ stands for density of the elastic medium,

With (1.3) in (1.5)

$A_{11}\frac{\partial^2 u_1}{\partial x_1^2} + (A_{12}+\mu_L)\frac{\partial^2 u_2}{\partial x_1 \partial x_2} + (A_{12}+\mu_L)\frac{\partial^2 u_3}{\partial x_1 \partial x_3} + \mu_L\frac{\partial^2 u_1}{\partial x_2^2} + \mu_L\frac{\partial^2 u_1}{\partial x_3^2} = \rho\frac{\partial^2 u_1}{\partial t^2}$,

$A_{22}\frac{\partial^2 u_2}{\partial x_2^2} + (A_{12}+\mu_L)\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + (A_{23}+\mu_T)\frac{\partial^2 u_3}{\partial x_1 \partial x_2} + \mu_L\frac{\partial^2 u_2}{\partial x_1^2} + \mu_L\frac{\partial^2 u_2}{\partial x_3^2} = \rho\frac{\partial^2 u_2}{\partial t^2}$,

$$A_{22}\frac{\partial^2 u_3}{\partial {x_3}^2}+(A_{23}+\mu_T)\frac{\partial^2 u_2}{\partial x_2 \partial x_3}+(A_{12}+\mu_L)\frac{\partial^2 u_1}{\partial x_1 \partial x_3}+\mu_L\frac{\partial^2 u_3}{\partial x_1^2}+\mu_T\frac{\partial^2 u_3}{\partial x_2^2}=\rho\frac{\partial^2 u_3}{\partial t^2},$$

………... (1.6)

When putting $\frac{\partial}{\partial x_3}$ =0, u_3=0 in first two equations of (1.6) we get deformation in the x_1x_2-plane as shown below

$A_{11}\frac{\partial^2 u}{\partial x^2}+B_2\frac{\partial^2 v}{\partial x \partial y}+B_1\frac{\partial^2 u}{\partial y^2}=\rho\frac{\partial^2 u}{\partial t^2}$,

$A_{22}\frac{\partial^2 v}{\partial y^2}+B_2\frac{\partial^2 u}{\partial x \partial y}+B_1\frac{\partial^2 v}{\partial x^2}=\rho\frac{\partial^2 v}{\partial t^2}$. .... (1.6 a)

The third equation of (1.6) is identically satisfied & we used thesymbolizationas

$X_1=x, X_2=y, u_1=u, u_2=v, B_1=\mu_L, B_2=A_{12}+\mu_L.$

1.2 Plane Wave in Infinite Media

We want to investigate the behavior of plane waves in xy-plane with displacements u in the direction of x-axis while displacement v in the direction of y-axis.

Here in the below figure the component of the plane wave are as:

1. ω is the circular frequency
2. θ is angle with the direction of plane wave transmission with y-axis

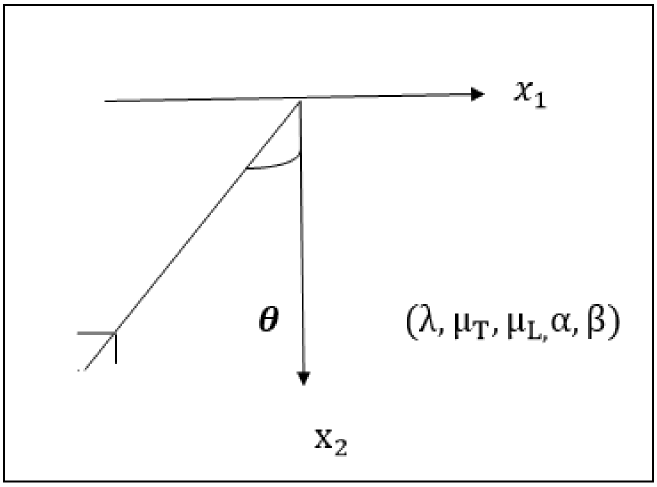

Fig 1.1

u=U exp (ip_1), v=V exp (ip_1),…………………… (1.7)

Substituting equation (1.7) in equation (1.1a),we have as in Singh and Singh(2004)

-(D_1-ρC^2)U +B_2sinθ $cos\theta$ V=0,

B_2sinθ $cos\theta$ U-(D_2-ρC^2)V=0 . ...(1.8)

$D_1$and$D_2$are given by

$D_1(\theta)$=$A_{11}sin^2\theta+B_1cos^2\theta$,

$D_2(\theta)$=$A_{22}cos^2\theta+B_1sin^2\theta$ .... (1.9)

The set of homogeneous equations (1.8) in U and V has a non –trivial solution only if

$|-(D_1-\rho C^2)B_2sin\theta cos\theta B_2 sin\theta cos\theta-(D_2-\rho C^2)|$, ... (1.10)

Outcomes of equation (1.10)

$2\rho C^2(\theta)$=$(D_1+D_2)\pm[(D_1-D_2)^2+4B_2^2sin^2\theta cos^2\theta]^{\frac{1}{2}}$ …....(1.11)

In equation (1.11) let's suppose C₁(θ) and C₂(θ) represents C in connection withwith above symbols.

Here $c_1$is the velocity of the quasi-P waves and $c_2$is the velocity of the quasi-S waves.

1.3 Propagation of Rayleigh Waves

In a Fiber reinforced anisotropic half-space the surface waves are transmitting in the direction of X-axis. Below are the components of Rayleigh waves:

1. ω refers to the circular frequency,
2. K refers to the wave number and
3. $C_R$refers to the phase velocity

We may assume result of (1.6a) as

u =U$e^{-kqy}\,e^{ik(C_Rt-x)}$,

v=$Ve^{-kqy}\,e^{ik(C_Rt-x)}$, ... (1.12)

q is assumed to be real and positive.Putting the values of the displacements in (1.6a),we get

($\rho {C_R}^2$ -A_{11}+B_1q^2)U + iqB_2V=0,

iqB_2U + ($\rho {C_R}^2$ -B_1+$A_{11}q^2$)V =0,.......................... (1.13)

The values of q may be obtained from

$\left|(\rho {C_R}^2 - A_{11} \ + B_1q^2) iqB_2 iqB_2 \rho {C_R}^2 - B_1 \ + A_{22}q^2\right|$ =0,

Which on simplification we get

$$q_1^2, q_2^2 \quad = \quad \frac{-\{(A_{22}+B_1)\rho C_R^2-A_{11}A_{22}-B_1^2+B_2^2\}\pm\left[\{(A_{22}+B_1)\rho C_R^2-A_{11}A_{22}-B_1^2+B_2^2\}^2-4B_1A_{22}(\rho C_R^2-A_{11})(\rho C_R^2-B_1)\right]^{\frac{1}{2}}}{2B_1A_{22}} \quad . \quad ....(1.14)$$

Therefore the solution (1.12) can be written as

u=$(U_{11}e^{-q_1ky}$+$U_{12}e^{-q_2ky})e^{-ik(C_Rt-x)}$,

v=$(V_{21}e^{-q_1ky}$+$V_{22}e^{-q_2ky})e^{-ik(C_Rt-x)}$, ... (1.15)

$(U_{11}, U_{12})$and (V_{21}, V_{22}) are not independent but are connected by (1.13) for q=$q_1$and q_2.

Taking second number of (1.13),we get

$\frac{U_{11}}{V_{21}} = m_1, \frac{U_{21}}{V_{22}} = m_2$, ...(1.16)

where $m_1 = iM_1$,$m_2 = iM_2$..........................(1.17)

$M_1 = \frac{\rho {C_R}^2 - B_1 + A_{22}{q_1}^2}{q_1B_2}$,......................(1.18)

and

$M_2 = \frac{\rho {C_R}^2 - B_1 + A_{22}{q_2}^2}{q_2B_2}$,........... .... (1.19)

where$q_1$and $q_2$are real and positive quantities defined in (1.14)

Therefore (1.15) becomes

u=$(m_1V_{21}e^{-q_1ky} + m_2V_{22}e^{-q_2ky})e^{ik(C_Rt-x)}$,

v$(V_{21}e^{-q_1ky} + V_{22}e^{-q_2ky})e^{ik(C_Rt-x)}$ . ...(1.20)

Boundary Conditions

The displacement in (1.20) must be true at the borderlinesituations,

$\sigma_{21} = \sigma_{22} = 0, at y = 0$, ... (1.21)

Where$\sigma_{21}$and $\sigma_{22}$are defined in (1.3).For plane deformation in the x_1x_2-plane ,$\frac{\partial}{\partial x_3} = 0$, u_3=0. Equation (1.3) then yields

$\sigma_{21} = \mu_L(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$,

$\sigma_{22} = A_{12}\frac{\partial u}{\partial x} + A_{22}\frac{\partial v}{\partial y}$, ...(1.22)

He we denote them as$x_1 = x, x_2 = y, u_1 = u$and $u_2 =$ v.

From equations (1.21) and (1.22) ,we get

$(m_1q_1 + i)V_{21} + \ (m_2q_2 + i)V_{22} = 0$,

$(A_{12}m_1i + A_{22}q_1)V_{21} + m_2A_{12}i + A_2q_2)V_{22}$=0. ....(1.23)

Eliminating $V_{21} and V_{22}$from (5.3) ,we get

$|m_1q_1 + \ im_2q_2 + iA_{12}m_1i + A_{22}q_1m_2A_{12}i + A_2q_2|$=0. ....(1.24)

On simplification, equation (1.24) becomes

$[(m_1q_1 + i)(m_2A_{12}i + A_{22}q_2) - (m_2q_2 + i)(A_{12}m_1i + A_{22}q_1)] =$
0. .... (1.25)

Here in a Fiber-reinforced elastic material thisabove equation is referred as equation of velocityof Rayleigh waves.

Particular Case

If we putα, β and $\left|\mu_L - \mu_T\right|$ equal to zero,the elastic coefficients become

$A_{11} = \lambda + 2\mu_T$, $A_{12} = \lambda$, $B_1 = \mu_T$,

$A_{22} = \lambda + 2\mu_T$, $A_{23} = \lambda$, $B_2 = \lambda + \mu_T$.................................. (1.21)

From equations (1.18) and (1.19) we get

$M_1 = q_1$,$M_2 = \frac{1}{q_2}$ then

${q_1}^2 = \left(1 - \frac{{C_R}^2}{\beta^2}\right), {q_2}^2 = \left(1 - \frac{{C_R}^2}{\alpha^2}\right)$, ... (1.27)

Where

$\alpha^2 = \frac{\lambda + 2\mu_T}{\rho}$And $\beta^2 = \frac{\mu_T}{\rho}$ . ... (1.28)

λ, $\mu_T$are Lame constants .Using equations (1.21)-(1.28), the velocity equation (1.25) reduces to

$(\frac{C_R^2}{\beta^2} - 2)^2 = 4(1 - \frac{C_R^2}{\alpha^2})^2(1 - \frac{C_R^2}{\beta^2})^2$ . ... (1.29)

1.4 Transmission of Love Waves

Consider an isotropic layer of Fiber-reinforced elastic medium of thickness H over a homogeneous anisotropic Fiber-reinforced elastic half space.

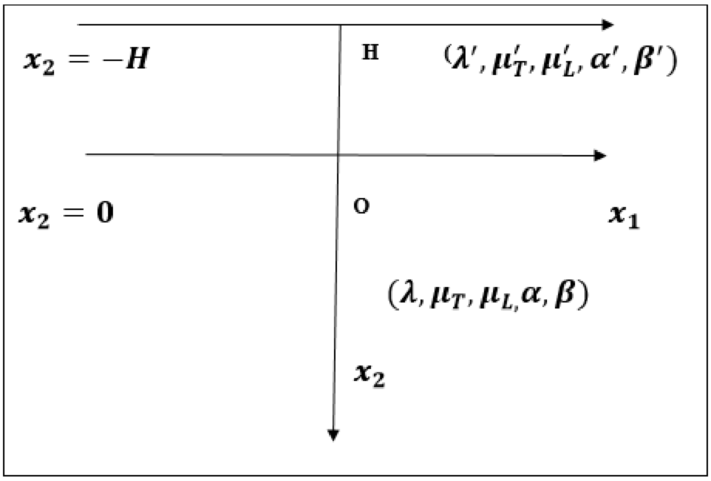

Fig. 1.2

Generally it is presumedthat the wave is transmitting towards the direction of planex_3.For antiplane strain in the upper layer motion's expression for Love waves can be derived from third equation of (1.1) by putting $u_{1=} u_2 = 0, u_3 = w, x_1 = x, x_2 = y, x_3 = z$ and $\frac{\partial}{\partial x_3}$=0, as

$$\mu_L' \frac{\partial^2 w'}{\partial x^2} + \mu_T' \frac{\partial^2 w'}{\partial y^2} = \rho' \frac{\partial^2 w'}{\partial t^2}, \qquad \text{.................................(1.30)}$$

wherep' is density and (μ_L', μ_T') are reinforced anisotropic elastic parameters respectively for the layer.The equation of motion for the half space is

$$\mu_L \frac{\partial^2 w}{\partial x^2} + \mu_L \frac{\partial^2 w}{\partial y^2} = \rho \frac{\partial^2 w}{\partial t^2}, \qquad \text{.................................... (1.31)}$$

Here in a half space the corresponding quantities are $\mu_L, \mu_T \; and \; \rho$

1.5 Solution of the Equation of the Motion

When a pllane harmonic wave is transmitting in the upper layer in the direction of x_3 plane we can presume the resolution of equation (1.30) expressed as

$$w' = W' e^{i\omega\left(t - \frac{x}{C_L}\right) - p'y}, \qquad \text{...................................(1.32)}$$

where$W'$is the amplitude factor ,$\omega and C_L$are circular frequency and phase velocity respectively in the layer.Substituting equation (1.32) in equation (1.30) we get

$$\frac{\omega^2}{C_L^2} \mu_L' - \mu_T' p'^2 = \rho' \omega^2. \qquad \text{.....................................(1.33)}$$

From equation (1.33), we can mention p'asp'=$\pm i p_1'$, where

$$p_1' = \frac{\omega}{C_L} \sqrt{\frac{\rho' C_L^2 - \mu_L'}{\mu_T'}} = {}^{K}\sqrt{\frac{\rho' C_L^2 - \mu_L'}{\mu_T'}}. \qquad \text{.....................................(1.34)}$$

Therefore equation (1.32) can be written as

$$w'(x, y, t) = W_1' e^{i\omega\left(t - \frac{x}{C_L}\right) - i p_1' y} + W_2' e^{i\omega\left(t - \frac{x}{C_L}\right) + i p_1' y}. \qquad \text{...................................... (1.35)}$$

Similarly the solution of equation (1.32) for half-space is

$$w(x, y, t) = W_1 e^{i\omega\left(t - \frac{x}{C_L}\right) - i p_1 y} + W_2 e^{i\omega\left(t - \frac{x}{C_L}\right) + i p_1 y}, \qquad \text{.................(1.36)}$$

where

$$p_1^2 = K^2 \left(\frac{\rho C_L^2 - \mu_L}{\mu_T}\right) \text{....................(1.37)}$$

$w(x, y, t) \rightarrow 0 as y \rightarrow \infty$.

Therefore $p_1 s$hould be purely imaginary.

Let

$$p_1 = i p_2 = i(K \sqrt{\frac{\mu_L - \rho C_L^2}{\mu_T}}). \qquad \text{..........(1.38)}$$

Equation (1.36) then show that

$$W_1 = 0. \qquad \text{...................................(1.39)}$$

Boundary Conditions

The below mentioned values of the components must be satisfied.

$$\tau'_{23} = 0 at y = -H,$$

$w'_3 = w_3 at y = 0$, ... (1.40)

$$\tau'_{23} = \tau_{23} at y = 0.$$

$\tau_{23}$is defined in (1.3) and can be written as (putting $u_2 = u_1 = 0,\ u_3 = w, x_1 = x, x_2 = y, x_3 = z$ and $\frac{\partial}{\partial x_3}$=0

$\tau_{23} = \mu_T \frac{\partial w}{\partial y}$;

Similarly $\tau'_{23} = \mu'_T \frac{\partial w'}{\partial y}$. ………………..(1.41)

From equations (1.31) – (1.41), we get

$$W'_1 e^{ip'_1 h} - W'_2 e^{-ip'_1 h} = 0,$$
$$W'_1 + W'_2 - W_2 = 0,$$

$\mu'_T p'_1 (W'_1 - W'_2) + \mu_T p_1 W_2 = 0.$ …………...(1.42)

Eliminating W'_1,$W'_2$and W_2 from equation (1.42) we obtain

$i tan\ (p'_1 H) = \frac{\mu_T p_1}{p'_1 \mu'_T}$. ……………..(1.43)

Substituting for p'_1 and$p_1$from (1.35) and (1.39) in (1.43),we get

$tan\left[\left(\frac{\rho' C_L^2 - \mu'_L}{\mu'_T}\right) KH\right] = \frac{(\mu_L - \rho C_L^2)^{\frac{1}{2}}}{(\frac{\rho' C_L^2 - \mu'_L}{\mu'_T})^{\frac{1}{2}}} . \frac{\mu_T}{\mu'_T}$ . ...(1.44)

In an anisotropic Fiber-reinforced elastic medium the above equation (1.44) refer to the Love wave's frequency equation.

Particular Case

If we put $\mu_L = \mu_T$,in (1.44), we get

$tan\left[\left(\frac{C_L^2}{\beta'^2} - 1\right)^{\frac{1}{2}} KH\right] = \frac{\mu_T}{\mu'_T} . \frac{(1 - \frac{C_L^2}{\beta^2})^{\frac{1}{2}}}{(\frac{C_L^2}{\beta'^2})^{\frac{1}{2}}}$, ...(1.45)

where$\beta'^2 = \frac{\mu'_T}{\rho'}$ and $\beta^2 = \frac{\mu_T}{\rho}$,

In a homogeneous elastic layer this above equation refers to the classical frequency of Love waves over a isotropic half space**.**

1.6 Propagation of Stoneley Waves

When Rayleigh waves are transmitting over the common boundary of two fiber-reinforced homogeneous elastic half spaces say $M' and\ M''$ we can generate Stoneley waves as shown in (**Fig**. 1.3)

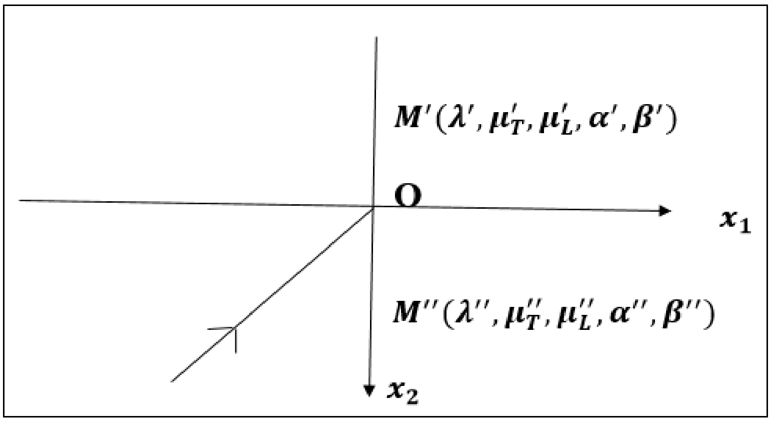

Fig. 1.3

The Stoneley wave's travels towards the mutual borderline of $M' and\ M''$in a fiber-reinforced elastic material.

The equation of motion for half space M'becomes

$A'_{11} \frac{\partial^2 u'}{\partial x^2} + B'_2 \frac{\partial^2 v'}{\partial x \partial y} + B'_1 \frac{\partial^2 u'}{\partial y^2} = \rho' \frac{\partial^2 u'}{\partial t^2}$,

$A'_{22} \frac{\partial^2 v'}{\partial x^2} + B'_2 \frac{\partial^2 u'}{\partial x \partial y} + B'_1 \frac{\partial^2 v'}{\partial y^2} = \rho' \frac{\partial^2 v'}{\partial t^2}$. ....(1.46)

Where $\rho'$is density and $(A'_{11}, A'_{22}, B'_1, B'_2)$ are Fiber-reinforced parameters for the upper half space. Similarly, the equations for lower half space $M''$are

$A''_{11} \frac{\partial^2 u''}{\partial x^2} + B''_2 \frac{\partial^2 v''}{\partial x \partial y} + B''_1 \frac{\partial^2 u''}{\partial y^2} = \rho'' \frac{\partial^2 u''}{\partial t^2}$,

$A''_{22} \frac{\partial^2 v''}{\partial x^2} + B'_2{}' \frac{\partial^2 u''}{\partial x \partial y} + B''_1 \frac{\partial^2 v''}{\partial y^2} = \rho'' \frac{\partial^2 v''}{\partial t^2}$. ....(1.47)

Where$\rho'$is density and $(A''_{11}, A''_{22}, B''_1, B'_2{}')$ refers to the quantities in lower half space.

We assume resolution for the equation (1.46) as

$$u' = \left(m'_1 V'_{21} e^{-q'_1 ky} + m'_2 V'_{22} e^{-q'_2 ky}\right) e^{ik(C_S t - x)},$$

$v' = \left(V'_{21} e^{-q'_1 ky} + V'_{22} e^{-q'_2 ky}\right) e^{ik(C_S t - x)}$, ... (1.48)

Where$m'_1 = iM'_1$,$m'_2 = iM'_2$, ... (1.49)

$M'_1 = \frac{\rho' {C_S}^2 - B'_1 + A'_{22} {q'_1}^2}{q'_1 B'_2}$,.................................(1.50)

and

$M'_2 = \frac{\rho' {C_S}^2 - B'_1 + A'_{22} {q'_2}^2}{q'_2 B'_2}$,....................................(1.51)

Where$q'_1 and q'_2$ are real and positive quantities defined in equatio?

$${q'_1}^2, {q'_2}^2 = \frac{-\left\{(A'_{22}+B'_1)\rho' {C_S}^2 - A'_{11}A'_{22} - {B'_1}^2 + {B'_2}^2\right\} \pm [\left\{(A'_{22}+B'_1)\rho' {C_S}^2 - A'_{11}A'_{22} - {B'_1}^2 + {B'_2}^2\right\}^2 - 4B'_1 A'_{22}(\rho' {C_S}^2 - A'_{11})(\rho' {C_S}^2 - B'_1)]^{\frac{1}{2}}}{2B'_1 A'_{22}} .$$

…………………….….. (1.52)

In M' medium the phase velocity of Stoneley wave's id denoted byC_S.

For lower medium $m''_1$and $m''_2$are defined as

$m''_1 = iM''_1$,$m''_2 = iM''_2$,

$$M''_1 = \frac{\rho'' {C_S}^2 - B''_1 + A''_{22} {q''_1}^2}{q''_1 B''_2},$$

And

$M''_2 = \frac{\rho'' {C_S}^2 - B''_1 + A''_{22} {q''_2}^2}{q''_2 B''_2}$, ... (1.53)

Where ${q''_1}^2$ and ${q''_2}^2$ are real and positive quantity defined in equation (1.53) by changing dash to double dash.C_SIs the phase velocity of stoneley waves in M''medium?

Similarly, the solution of (1.48) can be written as

$u'' = \left(m''_1 V''_{21} e^{-q''_1 ky} + m''_2 V''_{22} e^{-q''_2 ky}\right) e^{ik(C_S t - x)}$,

$v'' = \left(V''_{21} e^{-q''_1 ky} + V''_{22} e^{-q''_2 ky}\right) e^{ik(C_S t - x)}$. .... (1.54)

Boundary Conditions

The displacements in (1.48) and (1.54) must satisfy the boundary conditions,

$\sigma'_{21} = \sigma''_{21}$,
$\sigma'_{22} = \sigma''_{22}$, at y=0 .... (1.55)
$u' = u''$,
$v' = v''$,

Where $\sigma'_{21} = \sigma'_L \left(\frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x}\right)$,

$\sigma'_{22} = A'_{12} \frac{\partial u'}{\partial y} + A'_{22} \frac{\partial v'}{\partial y}$, ...(1.56)

$\sigma''_{21} = \mu''_L \left(\frac{\partial u''}{\partial y} + \frac{\partial v''}{\partial x}\right)$,

$\sigma''_{22} = A''_{12} \frac{\partial u'}{\partial y} + A''_{22} \frac{\partial v'}{\partial y}$,

Using equations (1.48) and (1.54) in equation (1.51), we get

$\mu'_L(m'_1 q'_1 + i)V'_{21} + \mu'_L(m'_2 q'_2 + i)V'_{22} - V''_{21}\mu''_L(m''_1 q''_1 + i) - \mu''_L(m''_2 q''_2 + i)V''_{22}$=0,
.... (1.57)

$(A'_{12} m'_1 i + A'_{22} q'_1)V'_{21}$ + $(m'_2 A'_{12} i + A'_{22} q'_2)V'_{22}$ - $(A''_{12} m''_1 i + A''_{22} q''_1)V''_{21} - (m''_2 A''_{12} i + A''_{22} q''_2)V''_{22} = 0$,
.... (1.58)

$m'_1 V'_{21} + m'_2 V'_{22} - m''_1 V''_{21} - m''_2 V''_{22}$ = 0 …………………....(1.59)

$V'_{21} + V'_{22} - V''_{21} - V''_{22} = 0,$ .... (1.60)

To conclude we acquired the expression for the velocity of the wave in the mutual borderline of medium $M' and M''$ by excluding $V'_{21}, V'_{22}, V''_{21} and V''_{22}$ from equations (1.57)-(1.60) as

$$\left|11 - 1 - 1m'_1 m'_2 - m''_1 - m''_2(m'_1 q'_1 + i)\mu'_L(m'_2 q'_2 + i)\mu'_L(-m''_1 q''_1 + i)\mu''_L(-m''_2 q''_2 + i)\mu''_L(A'_{12} m'_1 i + A'_{22} q'_1)(m'_2 A'_{12} i + A'_{22} q'_2) - (A''_{12} m''_1 i + A''_{22} q''_1) - (m''_2 A''_{12} i + A''_{22} q''_2)\right|$$

= 0 .... (1.61)

PARTICULAR CASE

If we put $\alpha', \beta'$and $|\mu'_L - \mu'_T|$ equal to zero, the elastic co-fficients for mediumM'become

$$A_{11}' = \lambda' + 2\mu_T' , A_{12}' = \lambda' , \qquad B_1 = \mu_{T,}'$$

$A_{22}' = \lambda' + 2\mu_T',\ A_{23} = \lambda',\ B_2 = \lambda' + \mu_T'$ '.........................(1.62)
From equations (1.50) and (1.51), we get
$M_1' = q_1' \quad ,M_2' = \frac{1}{q_2'}$, ...(1.63)
$q_1'^2 = \left(1 - \frac{C_S'^2}{\beta'^2}\right), q_2'^2 = \left(1 - \frac{C_S'^2}{\alpha'^2}\right)$,.......................(1.64)
where
$\alpha'^2 = \frac{\lambda' + 2\mu_T'}{\rho'}And\beta'^2 = \frac{\mu_T'}{\rho'}$ . ... (1.65)
λ', μ_T'Are Lame constants for upper half-space M' ? Similarly the quantities defined in
Equations (1.62)-(1.65) by changing dash to double dash show for lower half space. Putting
These values in equation (1.61), we acquire the Stoneley wave velocity equation in homogenous isotropic elastic half-spaces.

1.7 Conclusion

It has been observer that in a Fiber-reinforced material the propagation of surface waves is influenced by the reinforced parameters. It is clear from equations (1.34) and (1.38) for the presence of Love waves that p_i should be imaginary and $p_1'$is real satisfied if

$(\frac{\mu_L'}{\rho'}) < C_L^2 < (\frac{\mu_L}{\rho})$. ………… (1.66)

Equation (1.66) depends upon the reinforced parameters $\mu_L and \mu_L'$.

At the end we reached to a conclusion that the velocity of the Rayleigh waves is comparatively higher in a Fiber-reinforced elastic medium as compared to that of isotropic media. It is clear from equation (1.66) that a Stoneley wave is affected by the reinforced parameters.

References

[1]. Borcherdt, R.D., 1977, Reflection and refraction of type-II SH waves in elastic and anelastic media, Bull. Seism. Soc. Am. 67, 43-67.

[2]. Chaudhary, Sushil & Kaushik, V.P. & Tomar, S.. (2004). Reflection/transmission of plane SH wave through a self-reinforced elastic layer between two half-spaces. Acta Geophysica Polonica. 52.

[3]. Chattopadhyay, A., and S. Chaudhury, 1995, Magnetoelastic shear waves in an infinite selfreinforced plate, Int. J. Num. Anal. Methods in Geomech. 19, 289-304.

[4]. Kaushik, V.P., and S.D. Chopra, 1980, Transmission and reflection of inhomogeneous plane SH waves at an interface between homogeneous and inhomogeneous visco-elastic half-spaces, Geophys. Res. Bull. 18, 111-119.

[5]. Kaushik, V.P., and S.D. Chopra, 1982, Reflection and transmission of general SH waves at the interface between two heterogeneous and homogeneous visco-elastic media, Geophys. Res. Bull. 20, 1-20.

[6]. Schoenberg, M., 1971, Transmission and reflection of plane waves at elastic-visco-elastic interface, Geophys. J.R. astr. Soc. 25, 35-47.