Volume-03
Issue-07
July-2018

ISSN: 2455-3085 (Online)
RESEARCH REVIEW International Journal of Multidisciplinary
www.rrjournals.com [UGC Listed Journal]

Some aspect of minimal μ-open and μ-closed Sets in GT spaces

*Rebati Mohan Roy

Department of Mathematics, Mathabhanga College, Cooch Behar, West Bengal, India

ARTICLE DETAILS

Article History
Published Online: 05 July 2018

Keywords
μ-open set, μ-closed set, maximal μ-open set, minimal μ-closed set, locally minimal μ-open set, locally minimal μ-closed set,

***Corresponding Author**
Email: roy_rebati@rediffmail.com

ABSTRACT

In this paper, we introduce the concepts of locally minimal μ-open (resp. locally minimal μ-closed) sets as a certain point in a generalized topological space and obtain their some properties.

1. Introduction

Császár [1] introduced the concept of generalized topological spaces noting down that semi-open sets [4] and preopen sets [5] are not closed under finite intersections. Let X be a nonempty set and μ be a sub-collection of the power set of X. Then μ is a generalized topology on X if ∅∈μ and μ is closed under unions. We write, GT (resp. GT space) to denote a generalized topology (resp. generalized topological space) for brevity. The members of μ are called the μ-open sets in X and the complement of a μ-open set is called a μ-closed set in X. The generalized interior of a subset A of X is the union of all μ-open sets contained in A and is denoted by i_μ (A). The generalized closure of a subset A of X is the intersection of all μ-closed sets containing A and is denoted by c_μ (A). It is easy to see that i_μ(A)= X-c_μ (X-A). By a proper μ-open set (resp. μ-closed set) of X, we mean a μ-open set G (resp. μ-closed set E) such that G≠∅ and G≠X if X is μ-open (resp. E≠X and E≠∅ if ∅ is μ-closed). Here A⊆B means A is a subset of B and A⊊B means A is a proper subset of B.

2. Maximal, minimal μ-open set (resp. μ-closed set):

We recall the following known definitions and results to make the article self sufficient as far as practical.

2.1. Definition (Roy and Sen [6]) : A proper μ-open set A of a GT space X is called a maximal μ-open set if there is no μopen set U(≠A,X) such that A⊂U⊂X.

2.2 . Definition (Roy and Sen [6]) : A proper μ-closed set E of a GT space X is called a minimal μ-closed set if there is no μ-closed set F(≠∅,E) such that ∅⊂F⊂E.

2.3. Definition (S. Al Ghour et al. [7]) : A proper μ-open set U of X is said to be a minimal μ-open set if the only proper μ-open set which is contained in U is U.

2,4. Definition (Mukharjee [8]) : A proper μ-closed set E in a GT space X is called a maximal μ-closed set if any μ-closed set which contains E is X or E.

2.5. Definition (Sarsak [9]) : A GT space X is called μ-T_1 if for any of distinct points x and y, there exists a μ-open set U of X containing x but not y and a μ-open set V of X containing y but not x.

2.6. Definition (Császár [3]) : A GT space X is μ-connected if X cannot be expressed as G∪H=X where G,H are disjoined μ-open sets. As usual, if X is not μ-connected then X is called μ-disconnected. So X is μ-disconnected if there exists two disjoint μ-open sets G, H such that G∪H=X.

2.7. Theorem (Roy and Sen [6]): If A is a maximal μ-open set and B is a μ-open set in a GT space X, then either A∪B=X or B⊂A. If B is also a maximal μ-open set distinct from A, then A∪B=X.

2.8. Theorem (Roy and Sen [6]): If F is a minimal μ-closed set and E is a μ-closed set in a GT space X, then either E∩F=∅ or F⊂E. If E is also a minimal μ-closed set distinct from F, then F∩E=∅.

2.9. Theorem (Mukharjee [8]): If U is a minimal μ-open set and W is a μ-open set such that U∩W is a μ-open set, then either U∩W=∅ or U⊂W. If W is also a minimal μ-open set distinct from U, then U∩W=∅.

2.10. Theorem (Mukharjee [8]): If E is a maximal μ-closed set and F is any μ-closed set in a GT space X such that E∪F is a μ-closed set, then either E∪F=X or F⊂E.

2.11. Theorem (Roy and Sen [6]): A proper μ-open set A in a GT space X is maximal μ-open iff X-A is minimal μ-closed in X. Similarly, we see that a proper μ-closed set A in a GT space X is a maximal μ-closed iff X- A is minimal μ-open in X.

2.12. Theorem (Mukharjee [8]): If H is a maximal μ-open set and G is a minimal μ-open set in a GT space X such that H∩G is a μ-open set, then either G⊂H or the space is μ-disconnected

3. Locally minimal μ-open and μ-closed set :

We now introduce the following.

3.1. Definition : A μ-open set U(≠X) in a GT space X and x∈ X is said to be a locally minimal μ-open set at x if the μ-open set U containing x and there is a μ-open set V satisfying x∈ V ⊂U implies U=V. We see that all proper μ-open sets in a GT space X should be locally minimal μ-open sets if it is satisfied the above definition.

3.2. Example: Let X=R and μ= {∅, R , (a,∞), [a, ∞)}. In the GT space X, [a, ∞) is an only one locally minimal μ-open set at a and [a, ∞) is not a minimal μ-open set because of (a, ∞) is a μ-open set in X contained in [a, ∞). Also we observed that (a, ∞) is a locally minimal μ -open set at any point in it but [a, ∞) is not locally minimal μ-open set in it other than a.

3.3. Theorem: A minimal μ -open set U in a GT space X. Then U is a locally minimal μ-open set at each of its points.

Proof: Suppose that, there be x∈ U such that U is not a locally minimal μ-open set at x. There is a μ-open V in X satisfying x∈ V ⊊ U. But this contradicts the minimality of U.

3.4. Theorem: Let a GT space X and x∈ X. If U and V are locally minimal μ-open sets at x, then U=V.

Proof : If possible, let U≠V. By the definition of locally minimal μ-open sets U and V, neither U⊊ V nor V ⊊ U. Then x ∈ U∩V ⊊ U as well as x ∈ U∩V ⊊ V. Which is contradicts the definition of U and V. Thus U=V.

3.5. Lemma: Let a GT space X and x∈ X. If U is a locally minimal μ-open at x and V is a μ-open set containing x, then U⊆V.

Proof : Obviously, U∩V is a μ-open set satisfying x∈ U∩V ⊆U. According to the locally minimal μ-open set U, we have U ∩V = U and hence U⊆V.

3.6. Theorem: In a GT space X and x∈ X. If U is locally minimal μ-open at x, then there is no nonempty μ-closed set E such that E⊆U-{x}.

Proof: Let E be a nonempty μ-closed satisfying E ⊆ U-{x}. Then x∉ E and E⊊U. Then by Lemma 3.5, U ⊆ X-E i, e., E⊆X-U. As U∩(X-U)= ∅ it follows that E⊊ U and E⊆X-U cannot satisfy simultaneously.

3.7. Theorem: In a GT space X and x∈ X. If U is a μ-open set which is both locally μ-open at x and maximal μ-open, then U is the only proper μ-open set in X containing x.

Proof: Let G be a proper μ-open set in X containing x. By the locally minimality of U at x we have U⊆G (by the Lemma 3.5). But again by the maximality of U we have G⊆U or G∪U=X (by the Theorem 2.7). Here we see that G∪U=X contradicts U⊆G. Hence G⊆U and U⊆G holds simultaneously and hence U=G.

3.8. Definition: A μ-closed set E(≠X) in a GT space X and x∈ X is said to be a locally minimal μ-closed set at x if the μ-open set E containing x and there is a μ-closed set F satisfying x∈ F ⊂E implies F=E.

If a GT space X satisfies μ-T_1 axiom then for each x∈ X, {x} is the locally minimal μ-closed set at x. Thus each minimal μ-closed set is the locally minimal μ-closed set at each x.

Analogous to Theorem [3.3], Theorem [3.4], Lemma [3.5], Theorem [3.6], and Theorem [3.7], we have Theorem [3.9], Theorem [3.10],Lemma [3.11], Theorem [3.12], and Theorem [3.13] respectively. The proofs of the theorem are omitted as proofs are very much similar to corresponding results already established.

3.9. Theorem: A minimal μ-closed set E in a GT space X. Then E is a locally minimal μ-closed set at each of its points.

3.10. Theorem: Let a GT space X and x ∈X. If E and F are locally minimal μ-closed sets at x, then E=F.

3.11. Lemma: Let a GT space X and x∈ X. If E is a locally minimal μ-closed at x and F is a μ-closed set containing x, then U⊆V.

3.12. Theorem : In a GT space X and x∈ X. If E is locally minimal μ-closed at x, then there is no nonempty μ-open set U such that U⊆E-{x}.

3.13.Theorem: In a GT space X and x∈ X. If E is a μ-closed set which is both locally μ-closed at x and maximal μ-closed, then E is the only proper μ-closed set in X containing x.

3.14. Theorem: In a GT space X and x∈ X. If U is a locally minimal μ-open at x and E is a minimal μ-closed set not containing x, then E ⊆X - U.

Proof: As X-U is μ-closed and E is minimal μ-closed, it fallow that either E∩X-U=∅ or E⊆X- U (by Theorem [2.2]. In this case E⊆X-U implies that E⊆U. Since x∉ X, E⊆U-{x} which contradicts the Theorem [3.13], Hence E ⊆ U.

3.15. Theorem: In a GT space X and x∈ X. If U and E are respectively locally minimal μ-open and locally minimal μ-closed sets at x with U≠E, the below statements are true.

i). If U⊈E then U∩E is not μ-open.

ii). If E⊈U then U∩E is not μ-closed.

Proof: i) Let x∈ U∩ E⊆U. If U⊈E, then locally minimality of U at x implies that U∩E = U and which implies that U⊆E.

ii) Let x∈ U∩E ⊆E. If U⊈E, then locally minimality of E at x implies that U∩E = E and which implies that E ⊆ U.

4. Conclusion

We observed that a minimal μ-open (resp. minimal μ-closed) set is a locally minimal μ-open (resp. locally minimal μ-closed) at each of its points.

Acknowledgement

Acknowledge the supporters and people involved in research process

References

1. Á. Császár, Generalized open sets in generalized topologies, Acta Math. Hungar. 106 (1-2) (2005), 351—357
2. Á. Császár, Extremally disconnected generalized topologies, Ann. Univ. Sci. Budapest. Eötvös Sect. Math. 47 (2004), 91--96.
3. A. Császár, γ-connected sets, Acta Math.Hungar 101 (4) (2003), 273--279.

4. N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly 70 (1963), 36--41.
5. A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, On precontinuous and weak precontinuous mapping, Proc. Math. Phy. Soc. Egypt 53 (1982), 47—53.
6. B. Roy and R. Sen, On maximal μ-open and μ-closed sets via Generalized topology, Acta Math. Hungar. 136 (4) (2012), 233--239.
7. S. Al Ghour, A. Al-Omari, T. Noiri, On homogeneity and homogeneity components in generalized topological spaces, Filomat 27 (6) (2013), 1097--1105.
8. A. Mukharjee, On maximality and minimality of μ-open and μ-closed sets, An. Univ. Oradea Fasc. Mat. 23 (2) (2016), 15--19.
9. M.S.Sarsak, Weak separation axioms in generalized topological spaces, Acta Math.Hungar.,131(1-2) (2011), 110—121.

