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A Study of Vectors and the Geometry of Space

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ABSTRACT

Space is a term that can refer to various phenomena in science, mathematics, and communications. This Research Group explored modern geometry and the concept of space. Although a divide between ancient and modern geometry can be framed in different ways, the most useful one may well be the emergence of the consideration of space itself as an object of geometrical investigation. Greek mathematics understood geometry as a study of straight lines, angles, circles and planes, or in more general terms as a science of figures conceived against an amorphous background space whose definition lies outside the limits of the theory. This understanding was superseded by a conception of space (and spaces, now plural for the first time) itself endowed with geometrical properties. The main concern of this new geometrical science is to characterize the structures and features of geometrical space in axioms and demonstration. Although it is quite clear that this revolution in geometry helped shape the scientific world such that contemporary mathematics remains incomprehensible without it, the questions of when, why and how this revolution took place, prior to this research, were still to some extent obscure.Space is usually thought to begin at the lowest altitude at which satellites can maintain orbits for a reasonable time without falling into the atmosphere. This is approximately 160 kilometers (100 miles) above the surface. Astronomers may speak of interplanetary space (the space between planets in our solar system), interstellar space (the space between stars in our galaxy), or intergalactic space (the space between galaxies in the universe). Some scientists believe that space extends infinitely far in all directions, while others believe that space is finite but unbounded, just as the 2-space surface of the earth has finite area yet no beginning nor end.

1. Introduction

Geometry (from the Ancient Greek: γεωμετρία; *geo-* "earth", - *metron* "measurement") is a branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space. A mathematician who works in the field of geometry is called a geometer.

Geometry arose independently in a number of early cultures as a practical way for dealing with lengths, areas, and volumes. Geometry began to see elements of formal mathematical science emerging in the West as early as the 6th century BC.[1] By the 3rd century BC, geometry was put into an axiomatic form by Euclid, whose treatment, Euclid's *Elements*, set a standard for many centuries to follow.[2]Geometry arose independently in India, with texts providing rules for geometric constructions appearing as early as the 3rd century BC.[3] Islamic scientists preserved Greek ideas and expanded on them during the Middle Ages.[4] By the early 17th century, geometry had been put on a solid analytic footing by mathematicians such as René Descartes and Pierre de Fermat. Since then, and into modern times, geometry has expanded into non-Euclidean geometry and manifolds, describing spaces that lie beyond the normal range of human experience.[5]

While geometry has evolved significantly throughout the years, there are some general concepts that are more or less fundamental to geometry. These include the concepts of points, lines, planes, surfaces, angles, and curves, as well as the more advanced notions of manifolds and topology or metric.[6]

Geometry has applications to many fields, including art, architecture, physics, as well as to other branches of mathematics.

Contemporary geometry has many subfields:

- Euclidean geometry is geometry in its classical sense. The mandatory educational curriculum of the majority of nations includes the study of points, lines, planes, angles, triangles, congruence, similarity, solid figures, circles, and analytic geometry.[7] Euclidean geometry also has applications in computer science, crystallography, and various branches of modern mathematics.
- Differential geometry uses techniques of calculus and linear algebra to study problems in geometry. It has applications in physics, including in general relativity.
- Topology is the field concerned with the properties of geometric objects that are unchanged by continuous mappings. In practice, this often means dealing with large-scale properties of spaces, such as connectedness and compactness.
- Convex geometry investigates convex shapes in the Euclidean space and its more abstract analogues, often using techniques of real analysis. It has close connections to convex

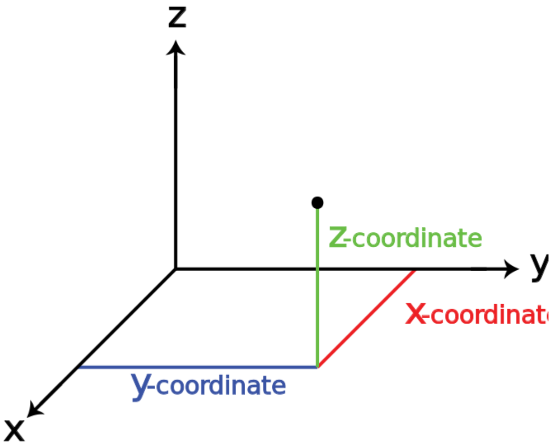
analysis, optimization and functional analysis and important applications in number theory.

- Algebraic geometry studies geometry through the use of multivariate polynomials and other algebraic techniques. It has applications in many areas, including cryptography and string theory.
- Discrete geometry is concerned mainly with questions of relative position of simple geometric objects, such as points, lines and circles. It shares many methods and principles with combinatorics.
- Computational geometry deals with algorithms and their implementations for manipulating geometrical objects. Although being a young area of geometry, it has many applications in computer vision, image processing, computer-aided design, medical imaging, etc.

Three-Dimensional Coordinate Systems

The three-dimensional coordinate system expresses a point in space with three parameters, often length, width and depth (xx, yy, and zz).

A three dimensional space has three geometric parameters: xx, yy, and zz. These are often referred to as length, width and depth. Each parameter is perpendicular to the other two, and cannot lie in the same plane. shows a Cartesian coordinate system that uses the parameters xx, yy, and zz.



Three-Dimensional Space: This is a three dimensional space represented by a Cartesian coordinate system.

Cartesian Geometry

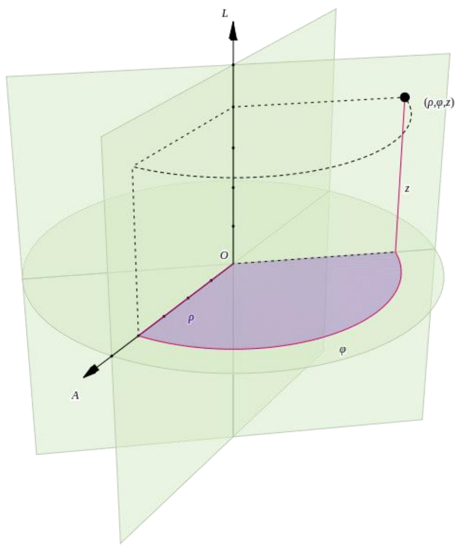
Also known as analytical geometry, this system is used to describe every point in three dimensional space in three parameters, each perpendicular to the other two at the origin. Each parameter is labeled relative to its axis with a quantitative representation of its distance from its plane of reference, which is determined by the other two parameter axes.

Other Coordinate Systems

Cylindrical Coordinates (ρ,φ,zρ,φ,z**)**

The cylindrical system uses two linear parameters and one radial parameter:

- ρρ*:* the radial distance from the point to zz
- φφ*:* the angle between the reference direction and the point
- zz*:* the distance from the reference plane to the point

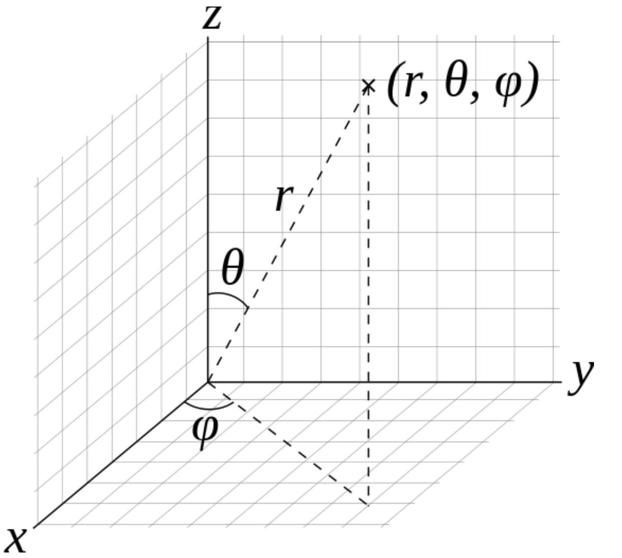


Cylindrical Coordinate System: The cylindrical coordinate system is like a mix between the spherical and Cartesian system, incorporating linear and radial parameters.

Spherical Coordinates (rr, θθ*,* φφ***)***

The spherical system is used commonly in mathematics and physics:

- rr: the radial distance from the origin to the point
- θθ*:* the angle between the zenith direction and directional vector of rr
- φφ*:* the angle from the reference direction to the orthogonal plane projected by the directional vector of rr



Spherical Coordinate System: The spherical system is used commonly in mathematics and physics and has variables of rr, θθ, and φφ.

Cartesian to Spherical

Often, you will need to be able to convert from spherical to Cartesian, or the other way around. The following equations will allow you to do just that:

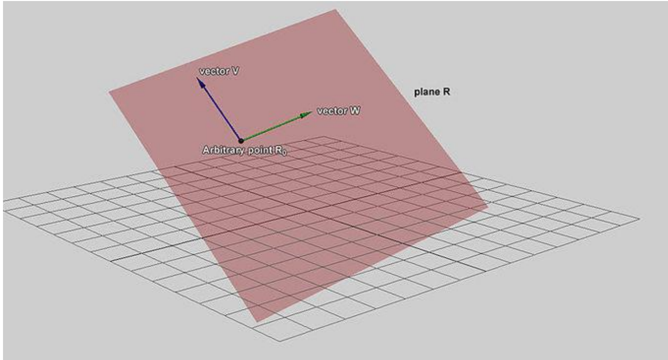
r=√x2+y2+z2r=x2+y2+z2

θ=arccos(zr)θ=arccos(zr)

φ=arctan(yx)

Vectors in the Plane

Planes in a three dimensional space can be described mathematically using a point in the plane and a vector to indicate its “inclination”.



Normal Vector to a Plane: This plane may be described parametrically as the set of all points of the formR=R0+sV+tWR=R0+sV+tW, where ss and tt range over all real numbers, VV and WW are given linearly independent vectors defining the plane, and R0R0 is the vector representing the position of an arbitrary (but fixed) point on the plane. The vectors VV and WW can be visualized as vectors starting at R0R0 and pointing in different directions along the plane. Note that VV and WW can be perpendicular but not parallel.

General form of the equation of the plane

In order to find the equation of the plane, consider the following: Let r0r0 be the position vector of some point P0=(x0,y0,z0)P0=(x0,y0,z0), and let n=(a,b,c)n=(a,b,c)be a nonzero vector.

The plane determined by this point and vector consists of those points PP, with position vector rr, such that the vector drawn from P0P0 to PP is perpendicular to nn. Recall that two vectors are perpendicular if and only if their dot product is zero. As such, the equation that describes the plane is given by:

n⋅(r−r0)=0n⋅(r−r0)=0

We can expand this equation in terms of its components to give:

a(x−x0)+b(y−y0)+c(z−z0)=0a(x−x0)+b(y−y0)+c(z−z0)=0

which we call the *point-normal* equation of the plane and is the general equation we use to describe the plane.

Vectors in Three Dimensions

A Euclidean vector is a geometric object that has magnitude (i.e. length) and direction.

A Euclidean vector (sometimes called a geometric or spatial vector, or—as here—simply a vector) is a geometric object that has magnitude (or length) and direction and can be added to other vectors according to vector algebra. A Euclidean vector is frequently represented by a line segment with a definite direction, or graphically as an arrow, connecting an initial point AA with a terminal point BB, and denoted by →ABAB→.

Vectors play an important role in physics: velocity and acceleration of a moving object and forces acting on it are all described by vectors. Many other physical quantities can be usefully thought of as vectors. The mathematical representation of a physical vector depends on the coordinate system used to describe it. Other vector-like objects that describe physical quantities and transform in a similar way under changes of the coordinate system include pseudovectors and tensors.

In the Cartesian coordinate system, a vector can be represented by identifying the coordinates of its initial and terminal point. For instance, in three dimensions, the points A=(1,0,0)A=(1,0,0) and B=(0,1,0)B=(0,1,0) in space determine the free vector →ABAB→ pointing from the point x=1x=1 on the xx-axis to the point y=1y=1 on the yy-axis.

Typically in Cartesian coordinates, one considers primarily bound vectors. A bound vector is determined by the coordinates of the terminal point, its initial point always having the coordinates of the origin O=(0,0,0)O=(0,0,0). Thus the bound vector represented by (1,0,0)(1,0,0)is a vector of unit length pointing from the origin along the positive xx-axis. The coordinate representation of vectors allows the algebraic features of vectors to be expressed in a convenient numerical fashion. For example, the sum of the vectors (1,2,3)(1,2,3) and (−2,0,4)(−2,0,4) is the vector:

(1,2,3)+(−2,0,4)=(1−2,2+0,3+4)=(−1,2,7)(1,2,3)+(−2,0,4)=(1−2,2+0,3+4)=(−1,2,7).

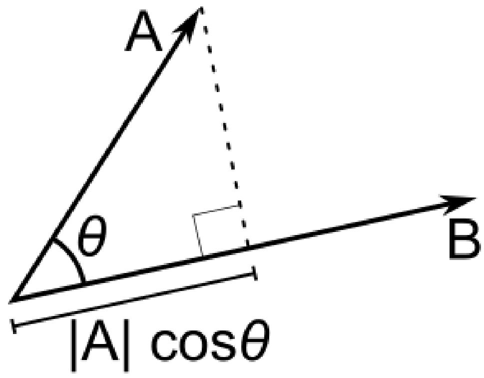
Vector in 3D Space: A vector in the 3D Cartesian space, showing the position of a point AA represented by a black arrow. ii, jj, and kk are unit vectors along the xx-, yy-, and zz-axes, respectively.

The Dot Product

The dot product takes two vectors of the same dimension and returns a single value.

The Dot Product

The dot product takes two vectors and returns a single value. The dot product can only be taken from two vectors of the same dimension. The dot product is the sum of the product of the corresponding parameters. Geometrically, the dot product is the product of the magnitudes of two vectors and the cosine of the angle between them. This is different from the cross product, which gives an answer in vector form.



Dot Product: When finding the dot product geometrically, you need the magnitudes of the vectors and the angle between them.

There are two representations of the dot product:

1. →a⋅→b=a1b1+a2b2+a3b3a→⋅b→=a1b1+a2b2+a3b3
2. →A⋅→B=|A||B|cosθA→⋅B→=|A||B|cos θ

Properties

Some of the properties of the dot product are

- The dot product is a commutative property, which means that the order of the terms does not change the outcome: →a·→b=→b·→aa→·b→=b→·a→
- The dot product is a distributive property: →a·(→b+→c)=→a·→b+→a·→ca→·(b→+c→)=a→·b→+a→·c→
- If two vectors are normal (perpendicular) to each other, their dot product will be equal to zero: →a·→b=0a→·b→=0

Example

Find the dot product of the two vectors →Q(5,2,8)Q→(5,2,8) and →R(6,−2,9)R→(6,−2,9):

→Q·→R=Q1R1+Q2R2+Q3R3=5·6+2·(−2)+8·9=98Q→·R→=Q1R1+Q2R2+Q3R3=5·6+2·(−2)+8·9=98

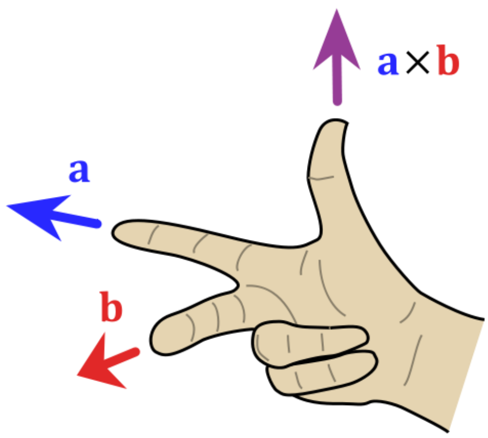
The Cross Product

The cross product is a binary operation of two three-dimensional vectors. The result is a vector which is perpendicular to both of the original vectors. Because it is perpendicular to both original vectors, the resulting vector is normal to the plane of the original vectors.

If the two original vectors are parallel to each other, the cross product will be zero.

The cross product is denoted as a×b=ca×b=c.

The direction of vector cc can be found by using the right hand rule.



The Right Hand Rule: If you use the rules shown in the figure, your thumb will be pointing in the direction of vector cc, the cross product of the vectors.

The magnitude of vector cc is equal to the area of the parallelogram made by the two original vectors.

The cross product is different from the dot product because the answer is in vector form in the same number of dimensions as the original two vectors, where the dot product is given in the form of a single quantity in one dimension.

The cross product can be found both algebraically and geometrically.

The geometric method of finding the cross product uses the magnitudes of the vectors and the sine of the angle between them:

a×b=|a||b|sinθa×b=|a||b|sinθ

The algebraic method of finding the cross product of two vectors involves inputting the vector information into matrices and manipulating them:

=[a2a3b2b3]i−[a1a3b1b3]j+[a1a2b1b2]k=∣∣∣ijka1a2a3b1b2b3∣∣∣=[a2a3b2b3]i−[a1a3b1b3]j+[a1a2b1b2]k=[ijka1a2a3b1b2b3]

The manipulated matrices form the following equations:

=<a2b3−a3b2,a3b1−a1b3,a1b2−a2b1>

2. Conclusion

A space consists of selected mathematical objects that are treated as points, and selected relationships between these points. The nature of the points can vary widely: for example, the points can be elements of a set, functions on another space, or subspaces of another space. It is the relationships that define the nature of the space. More precisely, isomorphic spaces are considered identical, where an isomorphism between two spaces is a one-to-one correspondence between their points that preserves the relationships. For example, the relationships between the points of a three-dimensional Euclidean space are uniquely determined by Euclid's axioms and all three-dimensional Euclidean spaces are considered identical.Topological notions such as continuity have natural definitions in every Euclidean space. However, topology does not distinguish straight lines from curved lines, and the relation between Euclidean and topological spaces is thus "forgetful". It is not always clear whether a given mathematical object should be considered as a geometric "space", or an algebraic "structure". A general definition of "structure", proposed by Bourbaki,[2] embraces all common types of spaces, provides a general definition of isomorphism, and justifies the transfer of properties between isomorphic structures.

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