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Significance of the Word 'Homogeneous' in Different Mathematical Contexts

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ABSTRACT

The word 'Homogeneous' play very important role in Mathematics. This word 'homogeneous' has different meanings in different mathematical contexts. We use it in ordinary differential equations, partial differential equations, linear algebra, probability and in many more mathematical areas. In this research paper, we will discuss how this word 'homogeneous' is used in mathematics.

1. Objective of the Study

This word 'homogeneous' has different meanings in different mathematical contexts. In Boundary value problems and Sturm-Liouville theory, a boundary value problem is called homogeneous if both the differential equation and boundary conditions are homogeneous. The word 'homogeneous has its significance in ordinary and partial differential equations also. An ordinary differential equation is that in which all the differential coefficients are with respect to a single independent variable. A partial differential equation is that in which all the differential coefficients are with respect to two or more independent variables. In probability, we often talk about homogeneous conditions .In algebra also, there are homogeneous equations. Moreover, In calculus, a function z is said to be homogeneous of x and y if it can be expressed in the form xnf(y/x). Hence there are many topics which can be discussed when we talk about the word 'homogeneous'. We are giving details of some mathematical contexts in which the word 'homogeneous' is used.

2. Boundary value problems

The general second order linear differential equation is

P(x)y''+Q(x)y'+R(x)y=G(x), (0<x<1 and P(x) is non zero) (1.1)

This equation is called homogeneous if G(x)=0 and non-homogeneous if G(x)≠0

The most general linear boundary condition at a point x0 is
$a_1y(x_0)+a_2y'(x_0)=c$ (1.2)

This boundary condition is said to be homogeneous if c=0 and non homogeneous if c≠0.

A boundary value problem is called homogeneous if both the equations (1.1) and (1.2) are homogeneous, otherwise it is said to be non-homogeneous.

Proposition 1: A boundary value problem y''+5y=0, y(-1)=0, y(1)=0 is homogeneous since both the differential equation and boundary conditions are homogeneous.

Proposition 2: A boundary value problem y''+6y=0 , y(0)=0, y(1)=1 is non-homogeneous since boundary conditions are non-homogeneous.

Proposition 3: A boundary value problem y''+6y=cos x, y(0)=0, y(1)=0 is non-homogeneous since differential equation is non-homogeneous.

3. Empirical Probability

The word 'homogeneous is also used in probability theory. 'According to Von Mises, "if an experiment is performed repeatedly under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the event occurs to the number of trials, as the number of trials becomes indefinitely large, is called the probability of happening of that event, it being assumed that the limit is finite and unique".

However, when the experimental conditions do not remain identically homogeneous, then the approach fails to determine the probability.

4. Partial derivatives and homogeneous function

In calculus, a function z is said to be homogeneous of x and y if it can be expressed in the form xnf(y/x). Here n is called degree of z.

Let $z= (x^{1/4}+y^{1/4})/(x^{1/5}-y^{1/5})$
$= \{x^{1/4}[1+(y/x)^{1/4}]\}/\{x^{1/5}[1-(y/x)^{1/5}]\}$
$= x^{1/20} f(y/x)$

Therefore z is homogeneous function with degree 1/20.

One of the Euler's theorems is also based on this function as:

If z be a homogeneous function of x and y of order n , then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$, for all x, y belongs to the domain of the function.

Use of the word 'homogeneous' in an Ordinary and partial differential equations

Equation of the form $x^n f(y/x)$ is also used in ordinary differential equations. If differential equation is of the form:

$$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)} \quad (1.3)$$

where f(x,y) and g(x,y) are homogeneous functions of same degree in x and y, then both of these function can be taken as:

$f(x,y)= x^n \gamma_1(y/x)$ and $g(x,y)= x^n \gamma_2(y/x)$

therefore (1.3) becomes $\frac{dy}{dx} = \gamma_1(y/x) / \gamma_2(y/x)$

Then we put y=vx, separate variables x and v , and then solve it.

There is one more type of differential equation in which the word 'homogeneous' is used:

The non-homogeneous linear differential equation of order n with constant coefficients is

$P_0 y^{(n)} + P_1 y^{(n-1)} + \ldots\ldots + P_n y = Q$, where P_0 , P_1 ,……………………,Pn are constants and Q is a function of x. Also P_0 is non-zero.

The corresponding homogeneous linear differential equation is

$P_0 y^{(n)} + P_1 y^{(n-1)} + \ldots\ldots + P_n y = 0$

But in partial differential equations, if all the derivatives occurring in linear partial differential equation are of same order n, then it is known a linear homogeneous partial differential equation of order n, otherwise it is called non-homogeneous.

For example: $(\partial^3 z/\partial x^3) + (\partial^3 z/\partial x^2 \partial y) + (\partial^3 z/\partial y^3) = 0$ is linear homogeneous equation as all the derivatives occurring in the equation are of same order 3.

But the equation $(\partial^3 z/\partial x^3) + (\partial^2 z/\partial y^2) = 0$ is non homogeneous as all the derivatives occurring in the equation are not of the same order.

5. Homogeneous Recurrence Equation

A recurrence relation of the form:

$c_0 a_r + c_1 a_{r-1} + c_2 a_{r-2} + \ldots\ldots + c_k a_{r-k} = f(r)$

where ci 's are constants is called a linear recurrence relation with constant coeffients. This recurrence relation is known as kth-order recurrence relation provided c0 and ck are non-zero.

A recurrence relation is called a linear non-homogeneous relation if it is of the form

$a_r + c_1 a_{r-1} + c_2 a_{r-2} + \ldots\ldots + c_k a_{r-k} = f(r)$,

where function f(r) is a function of 'r' or a constant and c_1, c_2,…….c_n are constants.The order of recurrence relation is r.

If f(n)=0, then recurrence relation is called homogeneous linear relation.

The recurrence relation : an+ 4an-1 + 11an-2 + 30an-3 = 0 is homogeneous , but if we replace 0 by any other non-zero term on right hand side, then this recurrence relation will be non- homogeneous.

6. Homogeneous Linear Equations in Linear Algebra

A general system of m linear equations with n unknowns can be written as:

$a_{11}x_1+a_{12}x_2+\ldots\ldots a_{1n}x_n=b_1$
a21x1+a22x2+………..$a_{2n}x_n=b_2$
………………………………
$a_{m1}x_1+a_{m2}x_2+\ldots\ldots a_{mn}x_n=b_m$,

where $x_1, x_2, \ldots, x_n$ are the unknowns, $a_{11}, a_{12}, \ldots\ldots a_{mn}$ are coefficients and $b_1, b_2, \ldots, b_m$ are the constant terms.

A system of linear equations is homogeneous if all of the constant terms are zero i.e.

$a_{11}x_1+a_{12}x_2+\ldots\ldots a_{1n}x_n= 0$
$a_{21}x_1+a_{22}x_2+\ldots\ldots a_{2n}x_n= 0$
………………………………
$a_{m1}x_1+a_{m2}x_2+\ldots\ldots a_{mn}x_n= 0$

A homogeneous system is equivalent to a matrix equation of the form

Ax=0

Where A is an m x n matrix, x is a column vector with n entries and 0 is the zero vector with m entries.

7. Results and Discussion

The word 'homogeneous' play very important role in many mathematical contexts. This word 'homogeneous' is used in linear algebra, partial differential equations , ordinary differential equations, probability theory, calculus, discrete and in many more mathematical fields. Even, only in different type of differential equations, meaning of this word 'homogeneous' is not same. It is very interesting fact.

8. Conclusion

The word 'homogeneous has its significance in any mathematical contexts'. The meaning of this word 'homogeneous' is not the same in different mathematical contexts.

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