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Probabilistic Metric Space and Fixed Point Theorem for Distinct Mapping

*Dr. Rajesh Vyas

Department of Mathematics, Christian Eminent College, Indore, MP, (India)

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***Corresponding Author**
Email: rajeshvyas09[at]rediff.com

ABSTRACT

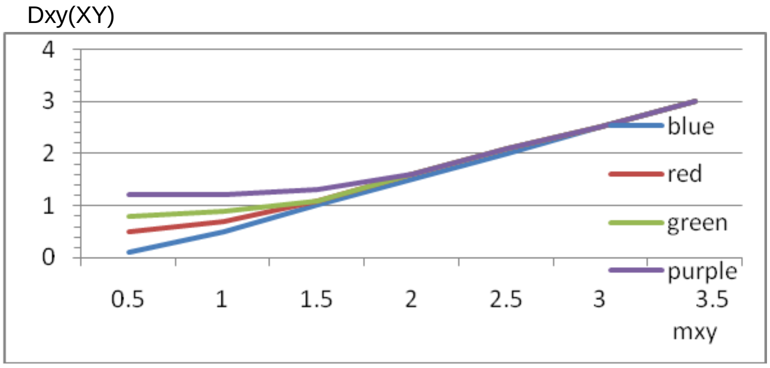
An active area of research is concerned with the study of fixed point theorems for mappings on metric spaces. The concept of an abstract metric space was first introduced by Frechet[1] in 1906 and is a natural setting for a large number of mathematical, physical and scientific problems in which the notion of distance appear. According to him "For any two points in the space, there is a single non negative real number with certain conditions called the distance between the points" is an essential feature of this concept. Perhaps, Brouwer gave the first result on fixed points is 1910[2,3]

1. Introduction

S. Gähler [4] framed an applicable system of axioms for a distance between three points and advanced a theory of 2- metric spaces. A slight expansion of the concept of 2- metric space was given in [3], where B. C. Dhage studied so called generalized metric spaces. Present paper is emphasised on our studies on contraction conditions for mappings that can be defined on a class of probabilistic metric space and fixed point theorems for distinct mappings.

Relations which involve chance are called probabilistic or stochastic relations. These are contrasted with deterministic relations, in which there is no element of chance. For example, Ohm's Law and Newton's Second Law involve no element of chance, so they are deterministic. However, measurements based on either of these laws do involve elements of chance, so relations between the measured quantities are probabilistic.As a particular cases we have obtain fixed point theorems for random operator and for mappings defined on deterministic metric spaces.

2. Application



Probability metric between two random variables X and Y, both having normal distributions and the same standard deviation (beginning with the bottom curve)

Definition 1.1: Let M be a non empty set and $d: M \times M \to R$, ,the set of real numbers. Then the pair (M,D)is called a metric space if 'd' satisfies the following axioms:

$M_1: d(p,q) = 0$ if p=q;
$M_2: d(p,q) \geq 0$;
$M_3: d(p,q) = d(p,q)$;
$M_4: d(p,r) \leq d(p,q) + d(q,r)$;

For all p, q, r in M. The mapping'd' is called 'metric' or 'distance function' on M.

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Definition 1.2: Let F be mapping of a set X into itself. An element $u \in X$ is called a fixed point of the mapping F if Fu= u.

By a fixed point theorem we shall understand a statement which assets that under certain conditions (on the mapping F and on the space X) a mapping F of X into itself admits one or more fixed points.

Brouwer's fixed point theorem involves a space X which is a topologically simple of R^n and mapping of X into itself, which is continuous, which asserts the existence of a fixed point whenever X is the unit ball in R^n and F is continuous.

In this theorem X can be replaced by any homeomarph thereof. This wasa first investigated by Birkhoff and Kellogg in 1912. Subsequently Schauder extended Brouwer's theorems to the case where X is a compact convex subset of a normed linear space. This theorem was extended to locally convex topological vector space by Tychonoff.

BANACH CONTRACTION PRINCIPLE: If T is a mapping of a complete metric space (X, d) into itself satisfying.

$$d(T_x, T_y) \leq kd(x, y)$$

For all x, y in X and for some $k, 0 \leq k < 1$, then T has a unique fixed point.

Later on Banach contraction principle were generalized and studied by many mathematicians for instance Fisher, Popa, S.L. Singh, Rhoade, Iseki, Jungck, Kasahara, Chang, Tarafdar& many others.

Fixed point theorems have extensive applications in proving the existence and uniqueness of the solutions of differential equations, Integral equations, Partial differential equations and in other related areas. Fixed point theorems have very fruitful applications in Eigen Value problems as well as in Boundary Value problems.

For numerous instances in which the theory of metric space is applied, this very association of a single number with a pair of elements is strictly speaking an over- idealization. In fact, it is appropriate to look upon the distance concept as a statistical or probabilistic rather than deterministic one, because the advantage of a probabilistic approach is that, it permits from the initial formulation a greater generality (and hence flexibility) than that offered by a deterministic approach.

In recent years e number of axiomatic systems has been evolved to grasp "uncertainties" in various physical systems. One of these is a Probabilistic metric space (PM- Space).

3. Conclusion

Karl Menger [6,7], who played an important role in the development of the theory of metric spaces, suggested that instead of associating a single non-negative number d(u, v)- the distance between two points u,v – a distribution function should associate with a pair of points. Thus for any points u, v in the space and for any x>0, we have a distribution function $F_{u,v}(X)$ and interpret $F_{u,v}(X)$ as the probability that the distance between u and v is less than x. This concept of a generalized metric space was introduced by Menger in 1942, under the name 'Statistical metric space'.[8].

The term 'Probabilistic metric space' was later adopted in 1964.

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