

# Bivalent Paradoxes As Fuzzy Midpoints

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## ABSTRACT

*This paper discusses paradoxes due to Cantor's definition of set and fuzzy paradoxes.*

## RUSSELL'S PARADOX

In 1908, Russell discovered a paradox known as Russell's Paradox. This paradox is directly arrived at from the definition of set by Cantor for any set  $M$  either  $M$  is an element of  $M$  or  $M$  is not an element of  $M$  i.e.  $M \in M$  holds or  $M \notin M$  holds.

Now consider the collections of consisting of all those sets  $x \in x$  holds.

Let

$Y = \{ x : x \in x \}$ . By the definition of Cantor,  $y$  is a set. Now as the question. Whether  $y \in y$ , or  $y \notin y$ ?

If  $y \in y$  holds, then  $y$  satisfies  $x \in x$

Therefore  $y \notin y$  .....(1)

If  $y \notin y$  is true then by the definition of  $y$ ,  $y$  is an element of  $y$ .

Therefore  $y \in y$  .....(2)

Hence in either case we have  $y \in y$  and  $y \notin y$  simultaneously which is a contradiction.

## PARADOXES OF GROUNDED SET

Definition of grounded set

Let a set  $x$  is called groundless if and only if there exists infinite sequences

of sets  $x_1, x_2, \dots, x_n$  (not necessarily all distinct) such that.....  $x_n \in \dots \in x_2, \in x_1$

A set which is not groundless is called grounded set.

Now let  $y$  be the collection of all grounded set. i.e.  $y = \{ x : x \text{ is grounded set} \}$ .

According to Cantor's definition of set  $y$  is a set. Now the question arises whether  $y$  is a grounded set or groundless set?

If  $y$  is grounded set, then  $y$  must be an element of  $y$ , as  $y$  contains all grounded set.

Hence  $y \in y$

Hence we have.....  $y \in y \in y \in y \dots \in y_2 \in y \in y$

This shows  $y$  is groundless.....(1)

If  $y$  is taken to be groundless set then there exists an infinite sequence of sets  $y_1, y_2, y_3, \dots, y_n$ . such that ,

.....  $y_n \in \dots \in y_2 \in y \in y$ ,

which shows that  $y$  is also groundless and  $y_1 \in y$ .....(2)

from (1) & (2)

We find a contradiction.

**PARADOX OF NON-CIRCULAR SETS**

A set x is called n-circular if there exists n-1 sets  $x_1, x_2, \dots, x_{n-1}$  such that

$$x \in x_{n-1} \in x_{n-2} \in \dots \in x_2 \in x_1 \in x$$

A set which is not circular is called non-circular set. Let y be collection of non circular sets i.e.

$$y = \{x : x \text{ is non - circular set}\}$$

Now question arises whether y is a circular set or non-circular set?

Let y be a circular set. Let it be n-circular set ( $n > 1$ ).

Then there exists sets  $y_1, y_2, \dots, y_{n-1}$  s-t,

$$y \in y_{n-1} \in y_{n-2} \in \dots \in y_2 \in y_1 \in y$$

Hence if  $n > 1$  we have,

$$\in y_2 \in y_1 \in y \dots \in y_2 \in y_1 \in y$$

Which shows that  $y_1$  is circular and  $y_1 \in y$  which is a contradiction as y contains non-circular sets only. If  $n = 1$ , we have  $y \in y$  which show that y contains a circular element y.

Which is a contradiction.

If y be non-circular set then y must be an element of y, as y is the collection of all non-circular sets. Hence we can write which shows that y is circular. Hence again we have contradiction.

**BURALI FORT'S PARADOX**

This paradox was published by Italian Mathematician Burali Forti in 1897 resulting well ordered set in  $\alpha$ .

Let W be the set of all ordinal numbers. W is a well ordered set in Cantor's Theory. Let  $\beta$  be the ordinal number of W. Let  $S(\beta)$  denote the set of all ordinal numbers less than. Now  $S(\beta)$  is an initial segment of W and the ordinal number of  $S(\beta)$  is  $\beta$ .

Here ordinal number  $S(\beta) =$  ordinal number of W. Hence W is similar to  $S(\beta)$  which is against a Cantor's theorem which says that a well ordered set is not similar to any of its initial segment. Thus the concept of set of all ordinal numbers leads to contradiction.

**CANTOR'S PARADOX**

This paradox was known to Cantor 1899 but was published in 1932.

Cantor proved that ordinal number of any Set M is less than the cardinal number of its power sets.

Let U be the set of all sets. Then every subset of U is also a member of U. Hence the power set of U is a subset of U. Hence the cardinal number of the power set of U is less than cardinal number of U which is against Canter's Theorem. Thus the concept of the set of all sets leads to contradiction.

Now we discuss fuzzy paradoxes.

**FUZZINESS AS MULTIVALANCE**

Fuzzy Theory holds that all things are matters of degree. In mechanizes much of our "Folk Psychology". Fuzzy Theory also reduces black-white logic and mathematics to special limiting cases of gray relationship. It violates black-white "Law of Logic", in particular the law of non-contradiction not – (A and not-A) and the law of excluded middle either A or not A and yet resolves the paradoxes or antinomies that these laws generate. Does the speaker tell the truth when he say he lies? Fuzziness also provides a fresh and deterministic, interpretation of probability and randomness.

Mathematically fuzziness means multivaluedness or multi valence and stems from the Heisenberg position momentum uncertainty principle in quantum mechanic. Three-valued fuzziness corresponds to truth, falsehood, and indeterminacy or to pre sense, absence and ambiguity. Multivalued fuzziness corresponds to degree of indeterminacy or ambiguity partial occurrence of events or relations.

**BIVALENT PARADOXES**

Now we discuss bivalent paradoxes and it will show that it is nothing but equivalent to fuzzy midpoints.

We consider the bivalent paradoxes as follows.

A California bumper sticker reads TRUST ME. Suppose instead a bumper sticker reads DON'T TRUST ME, should we trust the driver? If we do then, as the bumper sticker instructs, we do not. But if we don't trust the driver, then again in accord with the bumper sticker, we do trust the driver.

**CLASSICAL LIAR PARADOX**

Let another paradox which is the classical liar paradox. Does the liar from Crete lies when he says that all Cretans are liars? If he lies, he tells the truth. If he tells the truth, he lies.

**RUSSEL'S BARBER PARADOX**

Russell's Barber is a man who lives in a town whose advertises his service with logo "I shave all and only, those men who don't shave themselves." Now the question arises who shaves the barber? If he shaves himself, then according to his logo he does not. If he does not, then according to his logo he does.

Consider the card that says on one side "The sentence on other side is true" and says on the other side "The sentence on the other side is false".

The paradoxes have the same form. A statement P and its negative not P have the same truth-value t(p) such that  $t(p) = t(\text{not-}p)$ .....(i)

The two statements are either true or both false. This violates the law of no contradiction and excluded middle. For bivalent truth tables remind us that negation reverses truth value.

Hence we have  $t(\text{not-}p) = 1-t(p)$ .....(ii)

From (i) we have  $t(p) = 1-t(p)$  .....(iii)

or  $2t(p) = 1$ .....(iv)

or  $t(p) = 1/2$  .....(v)

If p is true, if  $t(p) = 1$ , Then

$1=0t(p)=0$  also implies

The contradiction  $1 = 0$

The fuzzy or multi valued interpretation accepts the logical relation (iii) and instead of insisting that  $t(p) = 0$  or  $t(p) = 1$ , simply solves for t(p) in (iii).

That is  $2 t(p) = 1$

or  $t(p) = \frac{1}{2}$

So the paradoxes reduce to literal half-truths.

Hence the "Paradoxes" reduce literal half-truths. They represent in the extreme the uncertainty inherent in every empirical statement and in many mathematical statements.

Geometrically the fuzzy approach places the paradoxes of the mid-point of the one-dimensional unit Hypercube [0,1].

More general paradoxes reside at the midpoint of n-dimensional hypercube, the unique point equidistant to all  $2^n$  vertices.

Multivaluedness also resolves the classical sorites paradoxes.

Consider a heap of sand is it still a heap if we remove one grain of sand? How about two grains? Three? If we argue bivalently by induction we eventually remove all grain and still conclude that a heap remains or that it has suddenly vanished. No single grain takes as from heap or non-heap.

Physically we experience degree of concurrence. In terms of statements about the physical processes we arrive again of degree of truth.

Suppose there are n grain of sand in the heap. Removing one grain leaves n-1 grains and a truth value  $t(S_{n-1})$  obeys  $t(S_{n-1}) < 1$  in general  $t(S_{n-1})$  may be close to unity, but we have some non-zero doubt  $d^{n-1}$  about the truth of the matter.

In 1983, Gaines established

$t(S_n) = 1-d_n$  .....(vi)

Where  $0 < d_n \leq d_{n-1} \leq \dots \leq d_{n-m} \leq \dots \leq 1$ . Hence  $t(S_{n-m})$

0 as  $m \rightarrow n$ . If we argue inductively, we can interpret overall inference as the forward chain.

"(If  $S_n$ , then  $S_{n-1}$ ), and (If  $S_{n-1}$ , then  $S_{n-2}$ ) and .....and (If  $S_1$ , then  $S_0$ )".

If we multiplicatively interpret the conjunction operator.

$t(S_n S_{n-m}) = (1-d_{n-k}) \quad k = 0 \text{ to, } k = n$  .....(vii)

If we interpret the conjunction operator as the minimum operator in the homework problems, then

$$t(S_n S_{n-m}) = \min (1d_n \dots \dots \dots 1d_{n-m}) \dots \dots \dots \text{(viii)}$$

$$= 1\max(d_n \dots \dots \dots d_{n-m}) \dots \dots \dots \text{(ix)}$$

In both cases the implication truth value  $t(S_n S_0)$  equals zero (or some small number). We pay a truth value fee, for each application of modus ponens of concluding B from A and AB. The overall inference is vacuous.

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